College Expenditures and Federal Aid Policy in the Market for Higher Education*

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Abstract

I extend the model developed by Epple et al. (2017) to include college expenditure choices and student preferences for quality of life on campus. In the extended model, colleges choose between two types of expenditures: (i) *educational expenditures*, which improve the college's *quality of education*, and (ii) *noneducational* or *luxury expenditures*, which improve the college's *quality of life*. This added dimension of competition between colleges allows them to specialize in their pursuit of new students: colleges can attract additional students by increasing their educational quality, but they can also attract new students by improving the quality of life on campus. This allows the model to test what has been coined the "Bennett Hypothesis"—the suggestion that increases in federal student aid lead to more luxurious college campuses (i.e., to higher noneducational expenditures).

To test the Bennett Hypothesis, a quantitative version of the model is specified and calibrated, borrowing heavily from the calibration performed in Epple et al. (2017). The calibrated version of the model does a very good job of matching aggregate characteristics of the U.S. market for higher education, including per-student *educational*

*This paper draws very heavily from the work of Dennis Epple, Richard Romano, Sinan Sarpça, and Holger Sieg (2017), and I am extremely grateful to each of them for their kindness not only in allowing me to use their work but also in helping me to understand it. Not only were they supportive of my attempt to extend their model of higher education, but they even went as far as to send me their code, which greatly improved my paper and saved me an enormous amount of time. I am particularly grateful to Sinan Sarpça who, when I struggled to understand some of the mathematics of their model, immediately sent me detailed notes explaining how to work it out. The work I present in this paper is only a tiny addition to their previous work, and the little amount I have managed to contribute was only made possible by their immense kindness and generosity.

and *noneducational* expenditures among private schools. After successfully calibrating the model so that it accurately reflects the U.S. market for higher education, an increase in federal student aid funding is simulated. While the increase in federal aid results in a general increase in enrollment and achievement, the model finds that bottom-tier private schools respond to the increase in aid by shifting their expenditures toward *noneducational* expenditures and away from *educational* expenditures, supporting the claims of the Bennett Hypothesis. Interestingly, this result is despite the fact that private schools are all modeled as *quality* maximizers rather than *profit* maximizers. That is, the bottom-tier private schools shift expenditures toward noneducational expenses in an attempt to attract more high quality students, choosing to target students with higher affinities for luxury. Perhaps most troubling from a policy perspective, students at the bottom-tier private schools are also the largest recipients of federal aid, suggesting a significant portion of the aid ends up being spent on noneducational expenses.

1 Introduction

Rising student loan debt, combined with the rising cost of college, has led to much concern regarding the accessibility of college. Despite significant increases in federal student aid funding during the Obama administration,[1] both student loan debt and the cost of college have continued to rise, spurring debate over the effectiveness of federal aid programs to increase affordability. Bennett and Wilezol (2013) argue that expanding federal aid programs simply allows colleges to charge higher prices. Originally put forth in 1987,[2] this has been coined the Bennett Hypothesis, and it has been one of the leading arguments used against the expansion of federal aid.

There is evidence to support the Bennett Hypothesis. Using a difference-in-differences approach, Lucca et al. (2018) find that tuition prices increase 60 cents for every dollar increase in the subsidized loan maximum, and 20 cents for every dollar increase in the unsubsidized loan maximum. Likewise, Turner (2014) finds that 12 percent of all Pell Grant aid is passed through to colleges.[3] However, pass-through effects of increased aid do not necessarily represent a failure of policy. If increases in aid lead to a larger number of students attending college, then the policy may be achieving its intended purpose. The extent to which student aid can be viewed as a success or failure depends on what the goal of the government is in implementing it.

In this paper, I consider three potential objectives of the government for implementing student aid policy. The first potential objective I consider is welfare maximization. While welfare maximization is often assumed to be the objective of the government, I believe that it is too broad of a goal in the context of student aid policy. Thus, the second potential objective I consider is that of maximizing educational attainment (i.e. maximizing the number of people who go to college). This goal seems more fitting, since increasing aid should

[1] The maximum Pell grant was $4,050 as the Obama administration began its first term, and increased to just under $6,000 by the end of its second term.

[2] Bennett, W. J. (1987). Our greedy colleges. *New York Times* 18, A27.

[3] These pass-through effects are estimated for public and private non-profit universities. There is evidence the pass-through effects are larger at private for-profit schools (Cellini and Goldin, 2014).

make college more affordable, allowing a greater number of students to attend. From this perspective, there is evidence that federal student aid policy has been successful.[4] Dynarski (2003) finds that increased aid not only has a significant impact on college attendance probabilities, but that it also increases the probability of a student *completing* school.[5] However, I believe that the objective of maximizing educational attainment is incomplete, since it doesn't account for the *quality* of education. Therefore, I believe the goal of the government is best represented by an objective to maximize aggregate achievement, where achievement depends on both attending college and the quality of the education provided by the college.

While increasing the number of people that attend college is likely viewed as a positive result by the government, focusing solely on this goal would ignore the potentially negative effects on the quality of the higher education system. For instance, if increased aid incentivizes students of lower ability to attend college, the overall quality of schools may decrease, potentially resulting in lower aggregate achievement despite an increase in enrollment. It is therefore necessary to consider the effect of student aid funding on the distribution of characteristics of students choosing to attend college, and not just the number of students that choose to attend.

Furthermore, how schools adjust their expenditures in response to changes in federal student aid funding is also of importance. If the aid that is passed through to colleges is used to improve student outcomes and the quality of their education, it would be difficult to view such pass-through effects as a failure of the policy. However, if increases in aid cause schools to focus more on frivolous or unnecessary expenditures which add little or nothing to the educational quality of the school (and possibly detract from it), this would likely be undesirable. Thus, it is also important to understand how federal aid policy affects the expenditure decisions of colleges. Given the magnitude of the effect that changes in federal

[4] For evidence that aid increases college enrollments, see Dynarski (2000), Stanley (2003), Kane (2007), and Cornwell et al. (2006). For reviews of the evidence, see Deming and Dynarski (2009) and Dynarski and Scott-Clayton (2013).

[5] This is important for maximizing educational attainment. If dropout rates increase, total educational attainment may decrease despite an increase in attendance rates.

aid policy create, such an analysis would need to be done through a general equilibrium framework. However, no theoretical model of higher education captures the alternative spending options available to schools.

Epple et al. (2017) develop a general equilibrium model of the market for undergraduate higher education that captures the coexistence of public and private colleges, the large degree of quality differentiation between them, and the tuition and admission policies that emerge from their competition for students. A quantitative version of their model does a very good job of matching aggregate characteristics of the U.S. market for higher education such as college attendance in public and private schools and tuition levels. However, their model does a less perfect job of matching the empirically observed provision of federal aid, and it says nothing about how colleges choose to spend their money. As will be demonstrated, extending their model to include college expenditures generates a better fit of the cross-sectional data, particularly with respect to the provision of federal aid. Moreover, the extended model generates new predictions about the market effects of an increase in federal aid.

In this paper, I build on the work of Epple et al. (2017) to develop a general equilibrium model of the US market for higher education that implements spending decisions among schools, providing insight into the ways in which schools are able to use expenditures to compete with one another for potential students. Using a quantitative specification of the model, I analyze the potential impact of federal student aid policy on college expenditures. I separate expenditures into two categories: (i) educational expenditures, which includes spending for any item that is directly related to the education of students, such as payments to instructors and construction and maintenance costs for lecture halls, and (ii) auxiliary expenditures, which includes spending for items that are not directly related to the education of students, such as dormitories, student union buildings, and sports stadiums. From hereon, I use the terms "auxiliary expenditures" and "luxury expenditures" interchangeably. Likewise, the terms "educational expenditures" and "instructional expenditures" are used interchangeably as well.

As in the model developed by Epple et al. (2017), the model I develop includes competing public and private colleges, federal aid modeled to approximate U.S. policy, and students that differ by income, ability, and unobserved idiosyncratic preferences for colleges. In addition, the model I develop includes a choice variable for each school for per-student luxury spending, and students also vary by their affinity for luxury.

To assess the impact of federal aid policy on college expenditures, a quantitative version of the model is specified. As in Epple et al. (2017), the quantitative model is calibrated for the 2007-08 academic year, allowing for direct comparisons between the performances and predictions of the two models. I find that the extended model does a better job of matching key characteristics of the market for higher education, and that it provides new insights into the ways in which schools compete with each other. Furthermore, the predictions of the quantitative model regarding the effects of an increase in federal financial aid are substantially different from those made by the model in Epple et al. (2017). Most notably, the model developed in this paper predicts much greater variation in the behavior of private schools.

This paper fills an important void in the existing literature on the economics of higher education. There have been theoretical papers that present models describing college pricing and admissions (Rothschild and White, 1995; Epple et al., 2006, 2017), but no model has yet considered the expenditure decisions of colleges. The model presented in this paper provides a structure under which the equilibrium effects of financial aid policy can be more thoroughly analyzed.

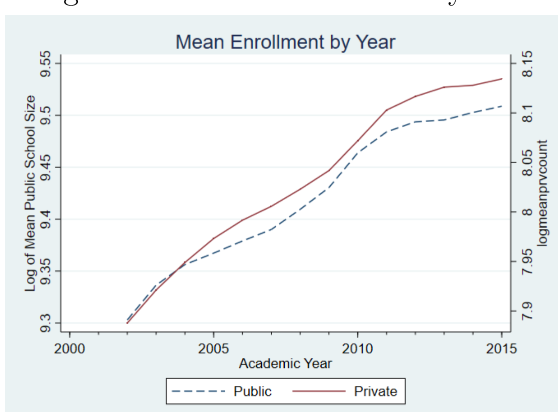
The remainder of this paper is organized as follows. Section 2 provides background information about college expenditures in the market for higher education, outlining the shortcomings of the model presented in Epple et al. (2017) and providing motivation for extending their model to include college expenditures. Section 3 presents the theoretical model. Section 4 presents a parameterization of the model and derives an equilibrium condition on college expenditures. Section 5 presents the quantitative model, demonstrates the model's ability to match key characteristics and empirical values of the market for higher education, and analyzes the effects of a change in the federal aid policy. Section 6 concludes. Appendix A provides proofs of the propositions, and Appendix B details the data and methods used.

2 College Expenditures

2.1 Time Trends

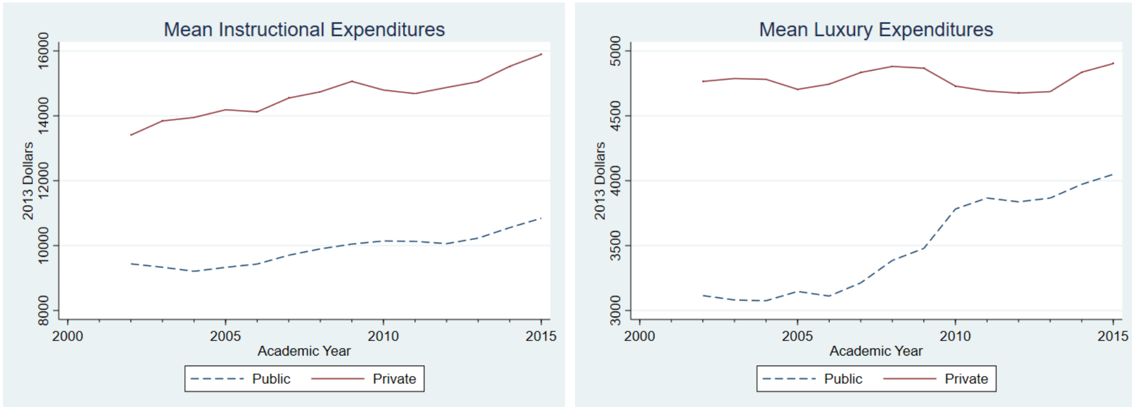
In the U.S. market for higher education, there is significant variation in the observed expenditures of schools. While average school size has been increasing at a very similar rate for both public and private schools, expenditures in the two sectors has not followed suit. Figure 1 shows the log of average student enrollment at public and private universities for academic years 2002-2015.[6] Figures 2 and 3 show the average per-student expenditures on instruction and luxury by year, respectively.[7]

Figure 1: Mean School Sizes by Year



[6]See Appendix B for details regarding the data used.

[7]All values are normalized to 2013 Dollars. Reported averages are weighted by the number of students at each school. Instructional expenditures are used as a proxy for educational expenditures. Auxiliary expenditures are used as a proxy for luxury expenditures. In both cases, per-student estimates are calculated using the reported FTE student count. See Appendix B for more information.



Figures 2 and 3: Mean Instructional and Luxury Expenditures by Year

Interestingly, growth in per-student instructional expenditures at private schools has outpaced that of public schools, while growth in per-student luxury expenditures shows the opposite relation. Moreover, the trends in per-student luxury expenditures for public and private schools diverge significantly following academic year 2007. Between 2007 and 2011, public school luxury expenditures increase dramatically, while private school expenditures decrease.

Around this time, there were two noteworthy factors affecting the market for higher education. The first was the financial crisis. The other was significant increases in federal aid funding.[8] The trends for public and private school luxury expenditures diverge the most between 2009 and 2010, which immediately follows the largest single-year increase in federal aid during the Obama administration. Although there are many factors that could have caused this divergence in spending trends between public and private schools, it will later be shown that the model presented in this paper recreates this observed spending pattern when an increase in federal aid is introduced.

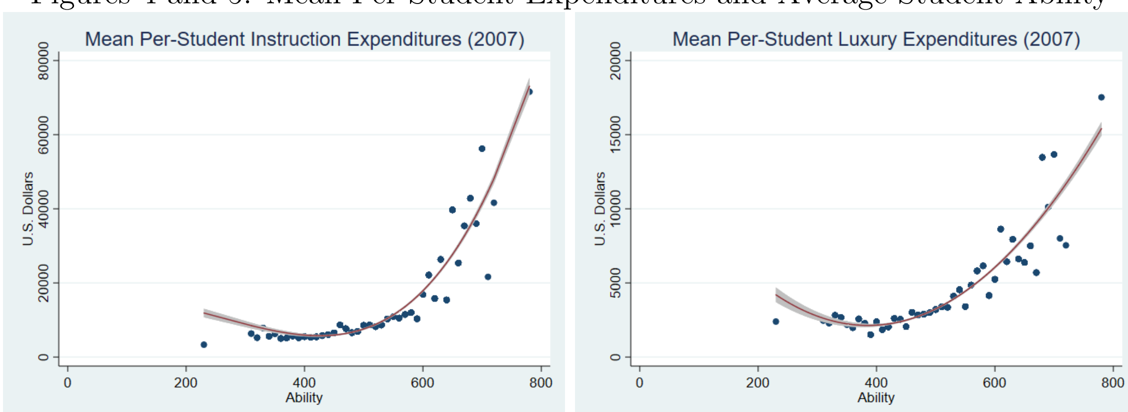
[8]The maximum Pell Grant award remained constant at $4,050 between academic years 2003 and 2006. The maximum was then increased to $4,310 for 2007, $4,731 for 2008, $5,350 for 2009, and then $5,550 for 2010 where it remained constant for several more years. The maximum and average award sizes for each year can be found at http://www.finaid.org/educators/pellgrant.phtml.

2.2 Cross Sectional Variation

Just as with prices and quality, there is significant variation across private schools regarding expenditures. As I will outline in this section, the ability to choose expenditures plays a fundamental role in the observed differentiation between schools.

While it might be expected that higher quality schools would have lower per-student expenditures on luxury than schools of lower quality,[9] this is in fact not the case. Figure 4 plots the average per-student instructional expenditures at private schools with the same average student ability.[10] Figure 5 plots average per-student expenditures on luxury. Note that only academic year 2007 is being considered.

Figures 4 and 5: Mean Per-Student Expenditures and Average Student Ability



As can be seen in Figures 4 and 5, both instructional and luxury expenditures increase dramatically as the average student ability at schools increases. The correlations between ability and per-student instructional and luxury expenditures are, respectively, 0.6588 and 0.5785.

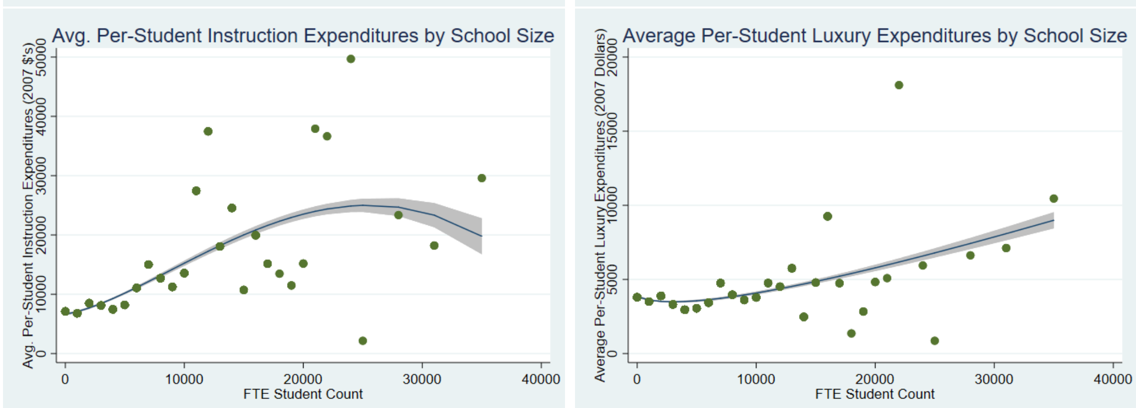
Similar patterns occur for per-student expenditures with respect to both tuition price and school size. As seen in Figures 6 and 7, per-student expenditures for both instruction

[9] Lower quality schools may try to attract students who have stronger preferences for luxury by increasing per-student expenditures on luxury. This would allow them to extract larger rents from such students.

[10] 25th percentile scores for the quantitative section of the SAT are used as a proxy for average student ability at each school. Each school's average SAT score is rounded to the nearest 10 point value (e.g. an average score of 627 becomes 630). Then the weighted average of per-student expenditures is calculated for the schools at each value of SAT score. See Appendix B for more detail.

and luxury tend to be greater at schools with higher tuition prices. Furthermore, per-student expenditures appear to grow exponentially with price, as they do with average student ability.

Figures 6 and 7: Mean Per-Student Expenditures by Tuition Price[11]



Figures 8 and 9: Mean Per-Student Expenditures by School Size[12]

However, Figures 8 and 9 show a much less clear relationship between per-student expenditures and school size. For the most part, per-student expenditures appear to be increasing in school size, at least for luxury expenditures. But the relationship between expenditures and school size is much less straightforward than for price or average student ability.

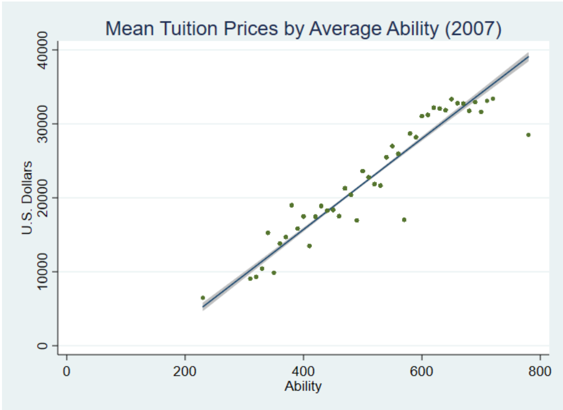
The strong relationship between per-student expenditures and both school quality and price is a reflection of the strong relationship between school quality and price. Figure 10 plots the relationship between average student ability at a school and the tuition charged.

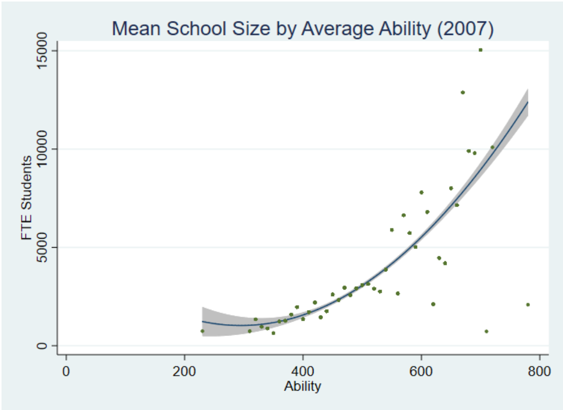
[11] Tuition prices are rounded to the nearest $1000. Each data point is generated by calculating the mean per-student expenditures for all schools with the same rounded tuition price, weighted by school size. See Appendix B for a more detailed description.

[12] The FTE student count for each school is rounded to the nearest 1000. Then, each data point is generated using the same approach used in the previous Figures. See footnotes 10 and 11, and Appendix B.

There is a clear linear relationship between the two. Higher quality schools have higher tuition prices.

Figures 10 and 11: Mean Tuition Prices and School Sizes by Average Student Ability

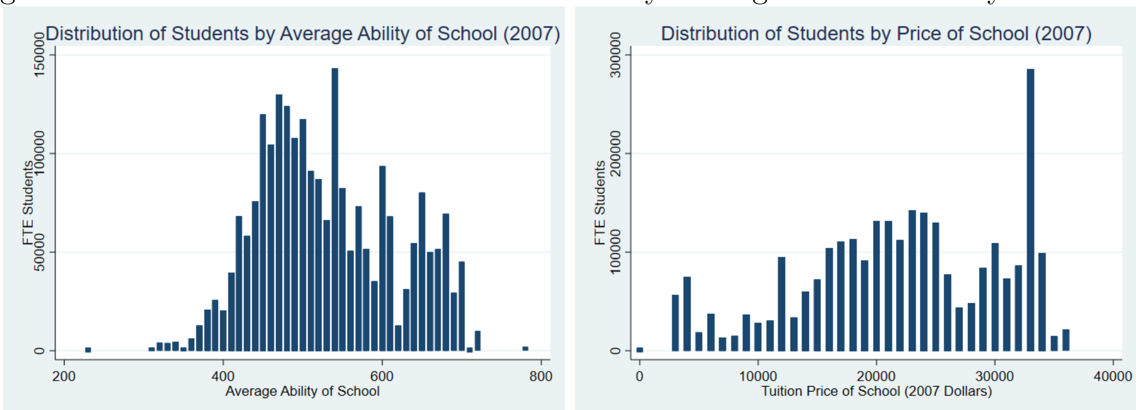




Interestingly, tuition prices increase linearly with average student ability, while per-student expenditures increase exponentially in ability and price. Thus, for the highest ability students who are selecting among the highest quality schools, marginal increases in price and ability lead to significant increases in per-student expenditures for both luxury and education. That is, the marginal returns of income and intelligence are greatest for those at the top of the distribution.

Figure 11 plots the relationship between school size and average student ability. On average, schools with higher average student ability tend to be larger. However, that is not to say that the majority of students attend the highest quality schools. While the size of schools increases as quality increases, the number of schools decreases substantially. Figure 12 shows the distribution of students by the average ability of the school they attend. Figure 13 shows the distribution of students by the price of tuition at the school they attend. In both cases, it is clear that the majority of students are not attending the highest quality schools.

Figures 12 and 13: Distributions of Attendance by Average Student Ability and Price[13]



To summarize, in the market for higher education we observe higher per-student expenditures for both luxury and instruction at more expensive, higher quality schools. Furthermore, the marginal increase in expenditures is largest for schools near the top of the quality distribution. Moreover, while these schools are on average larger, there are fewer of them, and they compete for a relatively small subset of the total student population.

The largest portion of students attends schools of medium quality and price. A smaller portion of students attends relatively lower quality schools where prices are also lower. Due to the linear relationship between price and average ability, the difference in price between the medium and low quality schools is about equal to the difference in price between the medium and high quality schools. However, because of the exponential relationship between per-student expenditures and average student ability, the difference in per-student expenditures between the low and medium quality schools is significantly smaller than the difference between the high and medium quality schools.

It will be shown that the model developed in this paper replicates these relationships between expenditures, quality, and price, providing additional insights into the workings of the market for higher education. Furthermore, the model presented in this paper is able to replicate the distribution of students across school qualities, placing the largest number of students in medium quality schools. This is a further advancement of the model developed

[13] See Appendix B for a description of how these Figures are derived.

by Epple et al. (2017), in which enrollment strictly decreases in quality and price.

By extending the model of Epple et al. (2017) to include colleges' expenditure decisions, the model now captures an important source of observed variation across schools. It is also now able to better replicate the observed attendance pattern across school qualities, as well as the observed distribution of federal student aid. Finally, when analyzing the effects of an increase in aid, the quantitative version of the model is able to replicate the observed divergence between public and private school expenditures that occurred during the Obama administration's expansion of federal aid.

3 Model

The model is based on the model developed in Epple et al. (2017). However, student preferences now include a term for luxuriousness, or quality of life, and schools can choose how much to spend on increasing the quality of life on their campus.

There are S states, each with one public university. In addition to the S public universities, there are R private universities, as well as an outside option to not attend school. Thus, there is a total of $J = S + R + 1$ alternatives for students to choose from. The total student population is normalized to be 1, with π_s denoting the proportion of the student population that lives within state $s \in S$,[14] such that $\sum_{s=1}^{S} \pi_s = 1$. Students differ continuously by after-tax income y, ability b, and their affinity for luxury z. Let $f_s(b, y, z)$ denote the density of (b, y, z) among students in state s.

[14]I am abusing notation here, using S to denote both the *set* of states as well as the *number* of states. I will similarly use J to denote both the set of all options as well as the total number of options. In all cases, meaning will be evident from context.

3.1 Federal Student Aid

Each school has a price cap P^c_{sj}, which is the tuition price listed by school j for a student from state s, and is the maximum amount that any student from state s can be charged.[15] The price cap for each school is assumed to be exogenous. Expected family contribution (EFC) is the estimated amount of funding a student is expected to receive from their family. For simplicity, it is assumed to only be a function of after-tax income y, and is denoted by $EFC(y)$. Cost of attendance (COA) at school j for a student from state s is given by $COA_{sj} = P^c_{sj} + M$, where M is the necessary allowance to cover living costs such as room and board. M is assumed to be the same for all schools.

A student's need is then calculated as the difference between their EFC and the COA. However, federal student aid is capped at a maximum award of amount $\overline{A}$. Thus, the federal student aid awarded to a student from state s at school j is given by

$$A_{sj}(y) = min\left\{max\left[0,\ P^c_{sj} + M - EFC(y)\right], \overline{A}\right\}. \tag{1}$$

3.2 Student Preferences

Student achievement is denoted by $a(q_j, b)$, where q_j is the educational quality of school j and b is the student's ability. Not attending a school provides an achievement of a_0. Student enjoyment is denoted by $e(l_j, z)$, where l_j is the quality of life at school j and z is the student's affinity for luxury. Not attending a school provides an enjoyment of e_0.

Schools observe a student's ability b and after-tax income y, but are unable to observe their affinity for luxury z. Thus, let $p_{sj}(b, y)$ denote the tuition charged by school j for a student from state s with ability b and after-tax income y.

Students' preferences contain idiosyncratic shocks for each school j, which are denoted by ε_j. These shocks capture differences in preferences that are unique to individual students

[15] Private schools list the same price caps for students from all states. That is, for each private school, $P^c_{sj} = P^c_j$ for all $s \in S$.

(e.g. a like or dislike for the local politics of a school, its climate, or its proximity to a major city). The preference shocks ε_j and the value of z are private information of each student.

The utility of student (b, y, z) for college j is assumed to be additively separable in the idiosyncratic component and given by:

$$U_j(s, b, y, z, \varepsilon_j) = \alpha U\left(y - p_{sj}(b, y) - M + A_{sj}(y), a(q_j, b), e(l_j, z)\right) + \varepsilon_j, \tag{2}$$

where $U(\cdot)$ is an increasing, twice differentiable, and quasi-concave function of the numeraire good money, educational achievement $a(\cdot)$, and enjoyment $e(\cdot)$; educational achievement is an increasing, twice differentiable, and strictly quasi-concave function of school quality and ability; enjoyment is an increasing, twice differentiable, and strictly quasi-concave function of school luxuriousness and affinity for luxury; and the term α is a weighting parameter.

Students choose the college j that maximizes their utility, subject to receiving admission. Whether or not a student (s, b, y, z) is given admission is discussed later in the section on schools' objectives. Let the optimal decision rule be denoted by $\delta(s, b, y, z, \varepsilon)$, where $\delta_j(s, b, y, z, \varepsilon) = 1$ if school j is chosen and 0 otherwise.

As in Epple et al. (2017), I assume that the vector ε satisfies the standard regularity assumptions in McFadden (1974) and integrate it out in order to derive conditional choice probabilities for each student type. The conditional choice probabilities are given by:

$$r_{sj}(b, y, z; P(s, b, y), A(s, y), M, Q, L) = \int \delta_j(s, b, y, z, \varepsilon) g(\varepsilon) d\varepsilon, \tag{3}$$

where $P(s, b, y)$ denotes the vector of tuition prices faced by a student of type (s, b, y), $A(s, y)$ denotes the vector of federal aid awards faced by a student of type (s, y), M denotes the vector of non-tuition costs (assumed to be the same at all schools), Q denotes the vector of college qualities, and L denotes the vector of college luxury levels.

3.3 School Objective

The objective of private schools is to maximize quality. The objective of state schools is to maximize the total student achievement of the in-state student population. In both cases, schools must ensure that profits remain nonnegative. I assume a cost function for school j of the form:

$$C(k_j, I_j, G_j) = F + V(k_j) + k_j I_j + k_j G_j,\ V', V'' > 0, \tag{4}$$

where k_j denotes the number of students enrolled at college j, I_j denotes per student expenditure on educational resources at school j, and G_j denotes per student expenditure on quality of living resources at school j. The costs $F + V(k_j)$ are independent of educational quality and luxuriousness, and they are referred to as the "custodial costs" (Epple et al., 2006).

School quality is given by $q_j(\theta_j, I_j)$, where θ_j denotes the mean ability of students enrolled at school j. School quality $q_j(\cdot)$ is assumed to be a twice differentiable, increasing, and strictly quasi-concave function of θ_j and I_j.

3.3.1 Private Colleges

The objective of private colleges is to maximize quality while maintaining non-negative profits. In addition to the revenue generated from tuition, private colleges are also assumed to have an exogenously determined endowment fund, denoted by E_j. It is further assumed that private colleges can be ranked by the size of their endowment, such that $E_1 < E_2 < ... < E_R$.

Taking the other schools' tuitions, qualities, and luxury levels as given, school j solves the following optimization problem:

$$\max_{\theta_j, I_j, G_j, k_j, p_{sj}(b,y)} q_j(\theta_j, I_j) \tag{5}$$

subject to the nonnegative profit constraint

$$\int\int\int\left(\sum_{s=1}^{S}\pi_s p_{sj}(b,y) r_{sj}(b,y,z;P(s,b,y),A(s,y),M,Q,L) f_s(b,y,z)\right) db\ dy\ dz \\ +E_j - F - V(k_j) - k_j I_j - k_j G_j \geq 0 \tag{6}$$

and the identity constraints

$$\theta_j = \frac{1}{k_j}\int\int\int\left(b\left[\sum_{s=1}^{S}\pi_s r_{sj}(b,y,z;P(s,b,y),A(s,y),M,Q,L) f_s(b,y,z)\right]\right) db\ dy\ dz, \text{ and} \tag{7}$$

$$k_j = \int\int\int\left(\sum_{s=1}^{S}\pi_s r_{sj}(b,y,z;P(s,b,y),A(s,y),M,Q,L) f_s(b,y,z)\right) db\ dy\ dz. \tag{8}$$

Solving this optimization problem, the following pricing strategy is obtained:

Proposition 1: *For any student (s,b,y) with $r_{sj} > 0$ for some z such that $f_s(z|b,y) > 0$,*[16] *school j will set tuition such that:*

$$p_{sj}(b,y) + \frac{\int r_{sj}(b,y,z;\cdot) f_s(z|b,y) dz}{\int \left(\partial r_{sj}(b,y,z;\cdot)/\partial p_{sj}(b,y)\right) f_s(z|b,y) dz} = V'(k_j) + I_j + G_j + \frac{\partial q(\theta_j, I_j)/\partial \theta}{\partial q(\theta_j, I_j)/\partial I}(\theta_j - b). \tag{9}$$

The proof of Proposition 1 is given in the appendix.

Note that this pricing strategy does not depend on the value of γ. That is, regardless of whether a school is a profit maximizer or a quality maximizer (or anything in between), it will employ the same pricing strategy. However, this does not imply that the school's optimal choice of inputs will not be affected. A quality maximizing school will set prices in order to

[16] If $r_{sj} = 0$ for all z, or if $r_{sj} > 0$ only for values of z such that $f_s(z|b,y) = 0$, the integral in the denominator of the second term will evaluate to zero.

maximize profits, thereby maximizing its ability to improve quality, but it will face a larger incentive to invest in educational expenses than a profit maximizing school. Also, since profit maximizing schools only care about quality so far as it affects their ability to charge higher prices, we would expect average student ability to be lower at profit maximizing schools as well. Furthermore, we would expect expenditure on luxuriousness to be higher at profit maximizing schools, where higher levels of luxury would allow them to extract larger rents from students who may be of lesser talent, but who have greater affinities for luxury.

This pricing strategy is very similar to the one derived in Epple et al. (2017), with two differences. The first difference is simply the addition of G_j on the right-hand side. The second difference is that in the fraction on the left-hand side the numerator and denominator are now being integrated over the conditional distribution of z, conditioned on the school's knowledge that the student is of type (s, b, y). That is, because the school can only observe s, b, and y, and not z, it takes the expectation of the probability that a student of type (s, b, y) selects their school, given their belief about the distribution of z among students of type (s, b, y). In the denominator, the expectation is taken for the marginal change in the probability of student (s, b, y) choosing school j with respect to a change in the price of school j. The conditional pdf of z, $f_s(z|b, y)$ allows for the possibility of dependency between z and b and y (e.g. if higher income students are more likely to have a greater affinity for luxury).

The right-hand side of (9) is the marginal cost of having student (s, b, y) attend the school, including the student's effect on the quality of the school, which is given by the last term. The last term is easier to interpret after making the following observations:

First, the marginal effect of student (s, b, y) on the average student ability is given by

$$\frac{\theta(k-1)+b}{k} - \theta = \frac{\theta k - \theta + b - \theta k}{k} = \frac{b-\theta}{k}. \tag{10}$$

Furthermore, the necessary change in I needed to keep quality the same after a change in θ

is given by $\frac{\partial q(\theta_j,I_j)/\partial\theta}{\partial q(\theta_j,I_j)/\partial I}$. Thus, since I is the cost per student, the total cost of maintaining the same level of quality after a change in θ is given by $\frac{\partial q(\theta_j,I_j)/\partial\theta}{\partial q(\theta_j,I_j)/\partial I}k_j$. Using these observations, the last term can be rewritten as

$$\frac{\partial q(\theta_j,I_j)/\partial\theta}{\partial q(\theta_j,I_j)/\partial I}(\theta_j-b)=\frac{\partial q(\theta_j,I_j)/\partial\theta}{\partial q(\theta_j,I_j)/\partial I}k_j\left(-\frac{(b-\theta_j)}{k_j}\right), \tag{11}$$

so that it is now easy to see that the last term is calculating the cost of maintaining the level of quality given the student's effect on average ability at the school. If a student is of type (b,y) with $b<\theta$, the last term in (9) will be positive, increasing the marginal cost of the student attending school j.

Henceforth, the right-hand side of (9) will be considered the effective marginal cost of the student for school j, denoted by

$$EMC_j(b)\equiv V'(k_j)+I_j+G_j+\frac{\partial q(\theta_j,I_j)/\partial\theta}{\partial q(\theta_j,I_j)/\partial I}(\theta_j-b). \tag{12}$$

$EMC_j(\cdot)$ is also a function of (k_j,I_j,G_j,θ_j), but this is suppressed to simplify notation. Note that for school j, EMC varies across students only by the student's ability.

The left-hand side of (9) demonstrates the school's ability to set prices above marginal costs. The probabilities must be nonnegative $[r_{sj}(b,y,z;\cdot)\geq 0]$ for all possible values of s, b, y, z, $P(s,b,y)$, $A(s,y)$, M, Q, and L, and, for any $r_{sj}>0$, $\frac{\partial r_{sj}(b,y,z;\cdot)}{\partial p_{sj}(b,y)}<0$. Therefore, the second term in (9) is always less than or equal to zero, resulting in price being set at or above marginal cost. Since the values of the integrals are determined by the distribution of z, schools are able to use investment in luxury items to increase their ability to charge prices above marginal costs.

3.3.2 Public Colleges

State schools are assumed to maximize the aggregate achievement of in-state students. Tuition prices for state schools are assumed to be exogenously set, with the in-state tuition denoted by T_s and the out-of-state tuition denoted by T_{so}. States also provide their state school with a per-student subsidy of w_s, financed by a balanced budget state income tax t_s.

Let $\gamma_s(b,y) \in [0,1]$ denote the fraction of in-state students of type (b,y) admitted by state school s, and let $r_{ss}(b,y,z;P,A,M,Q,L) \in [0,1]$ be the fraction of those admitted that choose to attend.[17] Likewise, let $\gamma_{so}(b,y) \in [0,1]$ denote the fraction of out-of-state students of type (b,y) the college admits, and let $r_{ts}(b,y,z;P,A,M,Q,L) \in [0,1]$ be the fraction of those admitted from state $t \neq s$ that choose to attend. Then the optimization problem for state school s is given by:

$$\max_{\theta_s, I_s, G_s, k_s, \gamma_s(b,y), \gamma_{so}(b,y)} \int\int\int \Big(a(q_s(\theta_s, I_s), b)\pi_s \gamma_s(b,y) r_{ss}(b,y,z;P,A,M,Q,L) f_s(b,y,z) \Big) db\ dy\ dz \tag{13}$$

subject to the budget constraint

$$\begin{aligned}
&\int\int\int \pi_s p_{ss}(b,y)\gamma_s(b,y) r_{ss}(b,y,z;P,A,M,Q,L) f_s(b,y,z) db\ dy\ dz \\
&\quad + \int\int\int \gamma_{so}(b,y) \left(\sum_{t\neq s} \pi_t p_{ts}(b,y) r_{ts}(b,y,z;P,A,M,Q,L) f_t(b,y,z) \right) db\ dy\ dz \\
&\qquad + w_s k_s - F - V(k_s) - k_s I_s - k_s G_s \geq 0,
\end{aligned} \tag{14}$$

the identity constraints

$$\begin{aligned}
\theta_s = &\frac{1}{k_s} \int\int\int b\pi_s \gamma_s(b,y) r_{ss}(b,y,z;P,A,M,Q,L) f_s(b,y,z) db\ dy\ dz \\
&+ \frac{1}{k_s} \int\int\int b\gamma_{so}(b,y) \left(\sum_{t\neq s} \pi_t r_{ts}(b,y,z;P,A,M,Q,L) f_t(b,y,z) \right) db\ dy\ dz
\end{aligned} \tag{15}$$

[17] Because schools cannot observe z, their admissions decisions γ_s can only depend on (b,y). However, schools know the probability with which students of type (b,y,z) will choose to attend.

and

$$k_s = \int\int\int \pi_s \gamma_s(b,y) r_{ss}(b,y,z;P,A,M,Q,L) f_s(b,y,z) db\ dy\ dz$$
$$+ \int\int\int \gamma_{so}(b,y) \left(\sum_{t\neq s} \pi_t r_{ts}(b,y,z;P,A,M,Q,L) f_t(b,y,z) \right) db\ dy\ dz, \tag{16}$$

the tuition regulation constraint

$$p_{ts}(b,y) = \begin{cases} T_s \text{ for all students } (t,b,y) \text{ with } t = s \\ T_{so} \text{ for all students } (t,b,y) \text{ with } t \neq s, \end{cases} \tag{17}$$

and the feasibility constraints

$$\gamma_s(b,y), \gamma_{so}(b,y) \in [0,1] \text{ for all students } (t,b,y). \tag{18}$$

The optimal policy of state schools is given by the following proposition:[18]

Proposition 2: *State school s admits all in-state students with $b \geq b^s_{min}$, all out-of-state students with $b \geq b^o_{min}$, and no other students, where*

$$a(q_s(\theta_s, I_s), b^s_{min})/\lambda + T_s + w_s - EMC_s(b^s_{min}) = 0, \text{ and} \tag{19}$$
$$T_{so} + w_s - EMC_s(b^o_{min}) = 0, \tag{20}$$

and λ is the multiplier on the budget constraint. Furthermore, since $EMC_s(b)$ is a decreasing function, it is implied that:

$$b^s_{min} (<)(=)(>) b^o_{min} \text{ as } a(q_s(\theta_s, I_s), b^s_{min})/\lambda + T_s (>)(=)(<) T_{so}. \tag{21}$$

[18] This result is exactly the same as the one derived in Epple et al. (2017). However, $EMC_s(b)$ is now different from that found in Epple et al. (2017). See Epple et al. (2017), Proposition 2.

Proof of Proposition 2 is given in the Appendix.

From equation (20), it is clear that no out-of-state student for whom $EMC_s(b)$ is greater than the revenue they generate will be accepted to the state school. However, from equation (19), we see that in-state students provide an additional marginal value to the state school, which consists of the student's contribution to the school's objective of maximizing in-state achievement. Thus, in-state students may be admitted even if their $EMC_s(b)$ is greater than the revenue they generate.

3.3.3 State and Federal Budgets Balance

It is assumed that per-student subsidies provided by states to their state schools are funded through state income taxes. Similarly, it is assumed that federal student aid is funded through a federal income tax. Furthermore, it is assumed that state and federal tax rates are set such that state and federal budgets balance.

Let Y_s denote aggregate pre-tax income in state s. The state income tax satisfies

$$t_s Y_s = w_s k_s \text{ for all } s \in S. \tag{22}$$

The federal income tax satisfies

$$t_f \left(\sum_{s \in S} Y_s \right) = \sum_{j \in J} \left[\int \int \int \left(\sum_{s=1}^{S} \pi_s r_{sj}(b, y, z; P, A, M, Q, L) A_{sj}(y) f_s(b, y, z) \right) db\ dy\ dz \right]. \tag{23}$$

4 Parameterizing the Model

In order to aid the analysis of each school's optimal choice of expenditures, I parameterize the model, closely following the parameterization in Epple et al. (2013). The quality function and luxury function are given, respectively, by

$$q_j = \upsilon_j \theta_j^{\tau} I_j^{\omega}, \quad \upsilon_j, \tau, \omega > 0, \quad \text{and} \tag{24}$$

$$l_j = G_j^{\delta}, \quad \delta > 0. \tag{25}$$

The achievement function and the enjoyment function are given, respectively, by

$$a(q_j, b) = q_j b^{\beta}, \quad \beta > 0, \quad \text{and} \tag{26}$$

$$e(l_j, z) = l_j z^{\phi}, \quad \phi > 0. \tag{27}$$

The utility function is given by

$$U_j\left(y - p_{sj} - M + A_{sj}(y), a_j, e_j\right) = \alpha \ln\left[\left(y - p_{sj} - M + A_{sj}(y)\right) q_j b^{\beta} l_j z^{\phi}\right] + \varepsilon_j. \tag{28}$$

The disturbances ε_j are assumed to be independent and identically distributed from a Type I Extreme Value Distribution. The location parameter is equal to zero and the scale parameter is equal to one. Therefore, student choices are modeled by a Logit model (Train, 2003), with choice probabilities for a student of type (s, b, y, z) given by

$$r_{sj}\left(b, y, z; P(s, b, y), A(s, y), M, Q, L\right) = \frac{\left[\left(y - p_{sj} - M + A_{sj}(y)\right) q_j b^{\beta} l_j z^{\phi}\right]^{\alpha}}{\sum_{k \in J} \left[\left(y - p_{sk} - M + A_{sk}(y)\right) q_k b^{\beta} l_k z^{\phi}\right]^{\alpha}}. \tag{29}$$

Note that the terms b^{β} and z^{ϕ} do not drop out of the probabilities, since the outside option is assumed to provide achievement of a_0 and enjoyment of e_0.[19] Furthermore, the price p_{sj}

[19] The outside option provides a utility of

$$U_0(y, a_0, e_0) = \alpha \ln\left(y a_0 e_0\right) + \varepsilon_0,$$

so that not all terms in the denominator of (29) contain b^{β} and z^{ϕ}. In this way, all choice probabilities depend on b and z in a nontrivial way.

that a school j can charge a student of type (s, b, y, z) is constrained by

$$p_{sj}(b, y) \leq y - M + A_{sj}(y), \tag{30}$$

so that the choice probabilities r_{sj} are between zero and one for all schools. That is, the maximum price a school can charge a student is the price at which the student's entire household income must be spent, and at that price the student's probability of choosing the school is zero.

With this parameterization of the model, the following condition can be found for the optimal expenditure on luxuriousness by school j:

Proposition 3: *For the parameterization given in (24)-(30), school j's optimal per-student expenditure on luxury can be expressed as*

$$G_j = \frac{\alpha\delta}{k_j} \int\int\int \left(\sum_{s=1}^{S} \pi_s \left[p_{sj} - EMC_j(b) \right] r_{sj}(1 - r_{sj}) f_s(b, y, z) \right) db\ dy\ dz, \tag{31}$$

if the school is a private school, and can be expressed as

$$\begin{aligned} G_s = \frac{\alpha\delta}{k_s} \Bigg[\frac{1}{\lambda} \int\int\int & a(q_s, b) r_{ss}(1 - r_{ss}) \pi_s \gamma_s f_s(b, y, z) db\ dy\ dz \\ & + \int\int\int \left[p_{ss} + w_s - EMC_s(b) \right] r_{ss}(1 - r_{ss}) \pi_s \gamma_s f_s(b, y, z) db\ dy\ dz \\ + \int\int\int \gamma_{so} & \left(\sum_{t \neq s} \left[p_{ts} + w_s - EMC_s(b) \right] r_{ts}(1 - r_{ts}) \pi_t f_t(b, y, z) db\ dy\ dz \right) \Bigg], \end{aligned} \tag{32}$$

if it is a state school, where λ is the multiplier on the budget constraint. Note that functional arguments have been suppressed.

The proof of Proposition 3 is given in the appendix.

Proposition 3 lends several insights for the expected expenditures on luxury in equilibrium. Looking at the expression for private school expenditures (31), we see that per-student expenditures at school j are decreasing in k_j, the number of students attending school j. Furthermore, the term $[p_{sj} - EMC_j(b)]$ in the integral gives the marginal value of each student type's attendance to the objective of school j (maximizing quality). Since this term is increasing in the price charged, per-student expenditures are increasing in prices. Moreover, because price caps place a limit on the maximum price a school is able to charge, luxury expenditures will likely be greater among schools with higher price caps, all else equal. However, if more expensive schools also have lower average student ability θ_j, then the term $EMC_j(b)$ may offset the positive effect of higher prices. In summary, we expect per-student expenditures on luxuriousness to be greatest at small, expensive private schools. Somewhat unintuitively, however, we also expect luxury expenditures to be greater among schools with higher ability students, all else equal.[20]

Looking at the expression for public schools (32), we see a very similar story to that of private schools. Per-student expenditures on luxury are decreasing with school size k_s. Expenditures are increasing with prices charged for in-state and out-of-state students, p_{ss} and p_{ts}, respectively. Moreover, state school expenditures are increasing in the size of the per-student subsidy w_s received by the school. As with private schools, per-student expenditures are expected to be greater among schools with higher average student abilities, all else equal. Unlike private schools, state schools wish to maximize aggregate in-state student achievement. The first term in brackets in expression (32) gives something comparable to the dollar value of total in-state student achievement in state s.[21] Thus, all else equal, we

[20] While unintuitive, this actually matches the data.

[21] The dollar value of total in-state student achievement at state school s is given by

$$\frac{1}{\lambda}\int\int\int a(q_s, b) r_{ss} \pi_s \gamma_s f_s(b, y, z) db\ dy\ dz,$$

since the integral gives the total achievement of in-state students at school s, and λ is the multiplier on the budget constraint. The first term in brackets in expression (32) has an additional term, $(1 - r_{ss})$. Thus, the term in brackets in expression (32) is strictly less than the dollar value of total in-state student achievement, but it will also be strictly increasing in the true dollar value. Therefore, all else equal, we expect per-student expenditures on luxury to be higher at state schools with greater dollar values of total in-state student achievement.

expect per-student expenditures on luxury to be higher among state schools with greater total in-state student achievement, and lower among state schools with greater values of λ.

Proposition 3 also provides insights into the expected effect that an increase in federal student aid will have on per-student luxury expenditures in equilibrium. Since an increase in aid will increase demand for college,[22] school sizes k_j should increase for all schools j.[23] This will have a downward effect on luxury expenditures for all schools, public and private. However, for private schools, the increase in school size will be offset by the school's ability to charge higher tuition prices in the face of increased demand. Since tuition prices are bounded by each schools price cap,[24] this effect will be greater for colleges with higher price caps. While it is not clear whether an increase in aid will result in more or less per-student luxury expenditures among private universities, (31) suggests that the effect will be larger for schools with higher price caps (e.g. if expenditures decrease for all private schools, they will decrease less at schools with higher price caps).

Since public schools are unable to increase tuition prices, their per-student luxury expenditures do not experience the same upward pressure as the expenditures of private schools. However, from (32), we can see that there are two ways in which G_s can be increased. First, the dollar value of total in-state student achievement can increase as a result of either an

[22] An increase in aid increases students' incomes (for students with incomes low enough to qualify), shifting the demand curve upward. It is possible that the increased demand among low income students could result in lower average abilities at schools, thus lowering demand for college. However, because private schools are quality maximizers, such an outcome is unlikely to occur, since quality is increasing in average student ability. Therefore, demand for all private schools should increase.

For public schools, average ability may decrease as more low income, low ability students are able to attend (low income students with high ability are likely to receive large scholarships from private schools, and thus not attend a state school). However, because state schools are interested in maximizing aggregate in-state student achievement, average student ability can only decrease if there is a large enough increase in either per-student educational expenditures I_s or total in-state student enrollment k_s. Since tuition prices for state schools are assumed to be fixed, state schools will likely be unable to offset the negative effect of a reduction in average student ability on aggregate in-state student achievement with an increase in I_s alone. Therefore, total student enrollment at state schools should increase. Thus, since prices at state schools do not change, demand for state schools should increase.

[23] In the short run, it is possible (and likely) that schools face capacity constraints that limit the potential increase in school sizes resulting from an increase in aid. However, such capacity constraints are not included in this model.

[24] It is possible that schools would choose to increase their price caps in response to an increase in federal student aid. However, I do not model this potentiality, and price caps are assumed to remain constant. It would be an interesting extension of the analysis to include a model for how schools choose their price caps.

increase in total in-state achievement or a decrease in λ (or both). This will increase the first term in brackets in (32). Second, an increase in demand from out-of-state students could result in a greater number of out-of-state students with higher ability. This will increase the third term in brackets in (32).

5 Quantitative Analysis

To better understand the effect that student aid policy has on schools' expenditures in equilibrium, I build a quantitative specification of the model. The model is calibrated for the 2007-08 academic year using the same calibration strategy implemented in Epple et al. (2017), so that direct comparisons can be made between their model and the model presented here. Furthermore, by calibrating the model for the 2007-08 school year, I am able to use estimates reported in Epple et al. (2017) that are estimated using confidential data on students' incomes and federal aid. Since I do not have access to these data, modeling the 2007-08 school year is necessary for calibration.

5.1 Calibration

The following must be calibrated: the number of colleges; in-state and out-of-state tuition prices and per-student subsidies at public universities; price caps and endowments of private universities; the college cost function; the joint income, ability, and affinity for luxury distribution of the potential student population; the federal aid formula; non-tuition costs; and the parameters of the utility, college quality, and college luxury functions. Many of these parameters can be directly calibrated using empirical observations. Given that this model is being calibrated for the 2007-08 school year, these parameter values should be the same as those reported in Epple et al. (2017). However, this approach cannot be used to calibrate the parameters of the utility, college quality, and college luxury functions. Instead, these parameters are calibrated such that the predictions of the model, regarding key market characteristics in equilibrium, approximate their empirical counterparts. I describe this strategy in more detail below. First, however, I outline the parameter calibrations taken from Epple et al. (2017).

Table 1 summarizes the parameter values used. The appendix provides additional information about the sources and methods used in calibrating the model.

5.1.1 Parameter Values Taken From Epple et al. (2017)

This paper wishes to investigate the effect of federal aid funding on college expenditures, as well as the impact that including expenditure decisions has on the performance of the model presented in Epple et al. (2017). In order to accomplish this latter goal, the model is specified such that it is as similar to the model presented in Epple et al. (2017) as possible. In this way, it can be argued that the differences in predictions made by the models are a result of the implementation of expenditure choices. Furthermore, by improving the model's ability to replicate empirically observed characteristics of the market for higher education, it can further be argued that the inclusion of college expenditure choices is an important addition to the model. Therefore, to the extent possible, the model is specified as it is in Epple et al. (2017).

First, following the work of Epple et al. (2017), the model is specified to have two states, with each state being identical with respect to its policies and distribution of student types, and each state having one state school. Furthermore, the model is specified to have three private schools. The price caps for these three schools are set equal to the price caps specified in Epple et al. (2017).

The following parameters are directly calibrated from empirical data for the 2007-08 academic year, and they are therefore set to be equal to the values reported in Epple et al. (2017): the average in-state and out-of-state tuition prices in 2007-08 for full-time undergraduates enrolled in public 4-year institutions were $6200 and $15,100, respectively. The average public subsidy per Full Time Equivalent (FTE) student was $8495. The average non-tuition cost of attending college (e.g. room and board, travel, books, supplies) was $10,250.

The EFC function is approximated using the EFC formula guide Worksheet A,[25] assuming the student is a dependent and is the only college student in a household with 3 or 4 family members. Furthermore, the student's income is set equal to zero, and the head of the household is assumed to be of age 45-54. The resulting empirical EFC function is piecewise linear with 7 adjusted income tiers. However, it is approximated by the following three tier function of after-tax income: $EFC(y) = max\{0,\ .48y - 10,300,\ .69y - 22,500\}$.

Federal student aid is measured as the weighted sum of grants, work-study aid, and loans using the following formula: Federal Aid = Grants + 0.33*Work-study + 0.1*Loan. The maximum Pell Grant in 2008 was $4731. The maximum subsidized federal loan amounts were $3500 and $4500 for the first two years of school, and $5500 for each year thereafter. The maximum possible work-study earnings were about $2500 on average (Epple et al., 2017). Plugging these values into the above formula provides an estimate of the maximum federal aid award very close to the $6000 assumed in the model.

Finally, I also follow the calibrations used in Epple et al. (2017) for the after-tax income and ability distributions of the student population. The after-tax income distribution is calibrated using the Current Population Survey (CPS) for 2009. It is specified as a lognormal distribution with a location parameter, given by: $\ln(y + 41,536) \sim N(11.46,\ .402)$. Ability is also calibrated assuming a lognormal distribution, and it is normalized such that $\ln(b) \sim N(1.0,\ 0.15)$, so that it matches the distribution of IQ. Following Epple and Romano (1998, 2008), the correlation between household income and student ability is set at 0.4.

[25] This worksheet can be found at https://studentaid.ed.gov/. The on-line appendix for Epple et al. (2017) describes the calibration of the EFC function in detail. A summary of their approach is given in the appendix to this paper.

Table 1: Parameter Values

Public School Parameters[a]	
T_s (In-state tuition)	6.2
T_{so} (Out-of-state tuition)	15.1
w_s (per-student subsidy)	8.5
Private College Parameters[a]	
Endowments	0.199, 0.543, 3.470
Price caps	22.5, 24.5, 32.5
Student Type Distribution Parameters	
b (Student ability)	$ln(b) \sim N(1.0,\ 0.15)$
y (Student household income)	$ln(y + 41,536) \sim N(11.46,\ 0.402)$
ρ (Ability-income correlation)	0.40
z (Student affinity for luxury)	$z \sim U(0,\ 1)$
Utility Function Parameters	
α	15.1
τ	0.15
ω	0.145
δ	0.0625
β	0.8
ϕ	0.35
υ_1, υ_2 (Efficiency parameters for state schools)	1
$\upsilon_3, \upsilon_4, \upsilon_5$ (Efficiency parameters for private schools)	1.025
a_0 (Outside option achievement)	1.72859
e_0 (Outside option enjoyment)	0.77202
M (Non-tuition costs[a])	10.25
Cost Function Parameters[a]	
F (Fixed costs)	0.24
υ_1	0.35
υ_2	30
Federal Aid Parameters	
$EFC(y)$	$max\,\{0,\ .48y - 10,300,\ .69y - 22,500\}$
$\overline{A}$ (Maximum aid amount[a])	6

[a] Measured in thousands of dollars.

5.1.2 Calibrating the Remaining Parameters

The following still need to be calibrated: the college cost function, the affinity for luxury distribution of the potential student population, and the parameters of the utility, college quality, and college luxury functions.

The affinity for luxury parameter z is assumed to be drawn from a uniform distribution with $z \sim U(0,1)$. It is also assumed to be uncorrelated with income and ability. The college cost function is specified as $C(k, I, G) = F + v_1 k + v_2 k^2 + kI + kG$. Thus, the following parameters of the cost function must be calibrated: F, v_1, and v_2.

The parameters of the utility, quality, and luxury functions include: α, τ, ω, β, v_j, δ, and ϕ. Furthermore, the achievement and enjoyment of the outside option, a_0 and e_0, must also be calibrated. These 9 parameters, along with the 3 parameters in the cost function, are calibrated such that the equilibrium approximates key aggregate characteristics of the U.S. market for higher education. In particular, the following empirical values are targeted: (i) the average private tuition net of institutional aid equal to \$23,400; (ii) the student percentage of private schools in total enrollment equal to 30; (iii) total college enrollment as a percentage of the potential student population equal to 40; (iv) the percentage of in-state students attending state colleges equal to 90; (v) the percentage of students at state and private schools receiving federal aid between 30 and 40; (vi) average federal aid received by students at private schools between \$2,000-2,500; (vii) average per-student expenditures on instruction at private schools equal to \$12,540; and (viii) average per-student expenditures on luxury at private schools equal to \$4,170. Refer to the first two columns of Table 2.

All 12 parameters jointly determine the predictions of the model in equilibrium. However, their influence over particular predictions varies across the parameters. The most significant effect of varying a_0 and e_0 is to alter the proportion of students that choose to attend college. The values of β and ϕ also affect the proportion of students that choose to attend college, since higher values of β and ϕ increase the payoffs to ability and affinity for luxury in college, respectively. The value of α mostly affects the proportion of in-state students at state schools. The relative values of υ_j between private schools and public schools mainly affects the proportion of students that choose to attend public schools. If υ_j is higher at private schools than at state schools, a greater proportion of students will choose to attend private schools. The relative values of τ, ω, and δ mainly affect the relative values of per-student expenditures on education and luxury, as well as affecting the average tuition and amount of financial aid received at private schools. Likewise, the parameters of the cost function are mainly identified by the average tuition, aid, and per-student expenditures at private schools. As costs increase, average tuition and aid increase and per-student expenditures decrease.

5.2 Baseline Equilibrium

The first two columns of Table 2 compare the output of the baseline model to the empirically observed data. The model does a very good job of matching the values used in calibration. In fact, the model does a significantly better job of matching the data than does the model developed in Epple et al. (2017),[26] providing some validation of the model and the importance of considering expenditures. The baseline model matches perfectly the total enrollment percentage, the percentage of students attending public schools, and the percentage of in-state students attending their state's school. The model's predictions for the average amount of federal aid received at private schools and the percentage of private school students receiving aid are both within the ranges specified in calibration. The greatest deviation from a value specified in calibration occurs for the average per-student instructional expenditures among private schools. However, even this value is very close to the one specified in calibration.

The model also does a good job of predicting values not used in calibration, albeit less precisely. The predicted percentage of public school students receiving federal aid is within

[26]See Epple et al. (2017), Table 3. The baseline specification of their model fails to accurately predict the proportion of in-state students attending their state's school, the average federal aid received at both public and private schools, the average tuition at state schools, and the percentages of students at both public and private schools receiving federal aid.

the empirically observed range, and the average tuition at state schools is matched exactly. However, state school expenditures are under-predicted, while average institutional aid at private schools and the average amount of federal aid received at public schools are both over-predicted. Extending the model to allow for greater variation among public schools could potentially resolve the model's difficulty with replicating the empirical values for state schools. That being said, overall the model fits the data very well, especially for private schools.

Table 2: Baseline Compared to Data and Aid Changes

	Data	Baseline	MaxAid=8	Change
Total Enrollment	40%	40.0%	42.5%	+6.25%
Share of Public School	70%	70.0%	70.1%	+0.14%
Proportion of In-State at State	90%	90.0	89.1%	-1.0%
Average Fed. Aid (State Schools)	1.25-1.5	1.89	2.82	+49.2%
Average Fed. Aid (Private Schools)	2-2.5	2.03	3.30	+62.6%
Average Institutional Aid	1.95	2.47	3.29	+33.2%
Average Private Tuition	23.40	23.35	22.47	-3.8%
Average State Tuition	7.09	7.09	7.17	+1.1%
Average I (State)	8.37	6.87	6.82	-0.7%
Average I (Private)	12.54	12.72	12.38	-2.7%
Average G (State)	2.77	2.46	2.42	-1.6%
Average G (Private)	4.17	4.21	3.90	-7.4%
Percent Receiving Fed. Aid (State)	30-40%	38.7%	43.4%	+12.1%
Percent Receiving Fed. Aid (Private)	30-40%	38.0%	46.6%	+22.6%
Average Student Cost (State)		15.45	14.60	-5.5%
Average Student Cost (Private)		31.58	29.42	-6.8%
Increase in Prv. Tuit. per $10,000 of Income	0-0.44	0.25	0.22	-12.0%
Decrease in Prv. Tuit. per 1 St.Dev. of Ability	0-2.39	0.39	0.25	-35.9%
Total Welfare		28.05	27.93	-0.43%
Total Achievement		2.130	2.144	+0.66%

Table 3 provides more detail for the baseline equilibrium. The first two rows report the values for the two identical state schools. These have been labeled A and B to emphasize their systematic differences from the private schools. The private schools have been numbered 3 through 5, ordered by the size of their endowments and price caps, and therefore also by their quality (q_j).[27] The private schools are much smaller (k_j) than the state schools. Mean student ability (θ_j), Instructional (I_j) and Luxury (G_j) expenditures per student, average tuition, average income, and the proportion of students paying full tuition all increase along the college quality hierarchy, just as they do in Epple et al. (2017) (with the exception of luxury expenditures, which are not included in their model).

Table 3: Baseline Values

j	k_j	θ_j	I_j	G_j	I_j/G_j	q_j	Ave.Tuit.	Ave.Inc.	Prop.Price Cap
A	0.1401	2.902	6.87	2.46	2.79	1.111	7.09	74.19	-
B	0.1401	2.902	6.87	2.46	2.79	1.111	7.09	74.19	-
3	0.0428	2.981	5.87	3.41	1.72	1.118	16.32	81.41	1.4%
4	0.0466	3.051	13.29	4.46	2.98	1.263	24.11	119.95	75.9%
5	0.0305	3.218	21.47	4.95	4.34	1.365	32.08	144.67	81.4%

j	Avg.Fed.Aid	% Rec.Aid	% Rec. $\overline{A}$	Avg.Aid Cond.	% Rec. $\overline{A}$ Cond.
A	1.89	38.7%	22.4%	4.89	57.9%
B	1.89	38.7%	22.4%	4.89	57.9%
3	3.59	64.6%	52.6%	5.55	81.4%
4	1.32	25.3%	17.4%	5.23	68.6%
5	0.90	17.5%	11.6%	5.17	66.3%

However, contrary to the baseline results in Epple et al. (2017), there is not an inverse relationship between school quality and size among the private schools. Instead, the medium quality school has the largest enrollment. As discussed in Section 2, there is a strong positive correlation between school size and average student ability (Figure 11), which suggests that we should see school sizes increasing along the college quality hierarchy in the model. However, the model does a much better job of matching the characteristics of the market for higher education if we instead view the schools in the model as representations of a par-

[27] Recall that private schools can be ranked by the size of their endowment, and that we have also calibrated their price caps to follow the same ranking (so that the school with the largest endowment also has the highest price cap). Since higher price caps allow schools the flexibility to charge higher prices (and provide their students with greater access to federal aid, since awards are calculated using price caps), and larger endowment funds provide schools with more money, schools with higher price caps and larger endowment funds face more relaxed budget constraints. Therefore, since private schools are assumed to be quality maximizers, school quality should follow the endowment/price cap hierarchy.

ticular *type* of school, and view their enrollment sizes as representations of the number of students that attend those types of schools. Viewing it in this way, the model predicts that the majority of private school students will attend schools of medium quality, with fewer students attending colleges of lower or higher qualities. As discussed in Section 2, this is in fact what is observed empirically (see Figure 12).

The bottom half of Table 3 shows the federal aid values produced in the baseline model. Average federal financial aid is about equal between public and private schools, but most of the financial aid among private schools is concentrated in the lowest quality school. Average federal aid is actually lower at the two higher quality private schools than it is at the state schools. Average federal aid declines as school quality increases, despite higher tuition prices, because much wealthier students attend the higher quality colleges.

The average income of students attending the lowest quality private school is not much higher than the average income at the state schools. Yet, those attending the private school pay more than twice as much for tuition on average. Interestingly, the private school spends less per-student on instructional expenditures than the state schools, but spends more per-student than the state schools for luxury expenditures. The private school is able to attract some high ability students by offering discounts on tuition, resulting in it having a higher average student ability (θ_j) than the state schools. However, because of its lower per-student expenditures on instruction, its quality is only slightly above that of the state schools. Thus, the fact that students are choosing to pay more than twice as much to attend the private school reveals a strong preference for luxury. Most worrisome from a policy perspective, federal financial aid is also highest at the lowest quality private school. Nearly two-thirds of its students receive aid, and more than 80% of those receiving aid receive the maximum award.

It is interesting to note that this behavior occurs under an objective of quality maximization. The lowest quality private school spends a proportionately greater amount than the other colleges on luxury expenditures in order to extract higher prices from students with greater preferences for luxury. But it does this so that it can increase revenue, allowing it to provide more discounts to high ability students who will raise its quality. As endowment funds and price caps increase, private colleges shift their expenditures to be proportionately more education intensive.

The baseline equilibrium shares most of the same characteristics as that of Epple et al. (2017). However, the implementation of expenditures into the model has resulted in a few significant differences. First, the distribution of private school students across the quality hierarchy of schools now has the largest number of students attending the medium quality school instead of the lowest quality school. As discussed above, I believe that this is a better representation of the true distribution of students in the private school market.

The quality distribution of private schools in the model has changed significantly as well. In the model developed in Epple et al. (2017), all three private schools are of much higher quality than the state schools. In this model, the lowest quality private school is nearly the same quality as the state schools, and it spends less per student than them on educational expenditures. I believe that this is a more realistic representation of the variation in schools that is present among private schools.

Finally, the model also differs in its ability to accurately replicate key values observed in the market for higher education, such as measures of financial aid, college expenditures, and the percentage of in-state students attending their state's school. The model's ability to better fit the values used in calibration, along with its more realistic representations of the observed distributions of students and college qualities, lend support for the predictions of the model.

5.3 Equilibrium Effect of an Increase in Aid

To test the model's predictions for the impact of an increase in federal financial aid, and to compare the differences in predictions between this model and the model developed in Epple et al. (2017), I increase the maximum federal aid award from \$6000 to \$8000. The aggregate effects of the policy change are summarized in Table 2. The new equilibrium values are reported in column 3, and the percentage changes from the baseline equilibrium are reported in column 4. Table 4 reports the new equilibrium values for each school.

Just as in Epple et al. (2017), total enrollment increases by about 6% of the initial college population, with more than two thirds of this increase occurring in public schools.[28] Furthermore, both models predict large increases in the average amounts of federal aid received at both public and private schools, as well as increases in the percentage of students receiving federal aid. Likewise, both models also predict a substantial increase in the average amount of institutional aid given at private schools, along with decreases in the average student costs at both public and private schools. While the aggregate effects of the increase in federal aid are fairly similar in both models, the effects for each school are very different.

Table 5 reports the percentage change in each value between the baseline equilibrium and the new equilibrium. Enrollments increase at every school, and the average income at each school decreases, as more low-income students are able to afford college. The average student ability at public schools decreases a little as they begin to let more out-of-state students of lower ability in, valuing their higher out-of-state tuition more than the negative impact on mean ability caused by their attendance. This shift towards out-of-state students increases the average tuition received at the state schools.

Tuition prices at the top two private schools remain the same, only increasing a half percent at the middle quality school. However, both schools are able to enroll more low income students of high ability, increasing the average ability at their schools as the average incomes of their students decrease. After the increase in federal aid, the top two private schools shift their spending towards education, decreasing their per-student luxury expenditures and increasing their per-student educational expenditures. The two state schools also shift their spending towards educational expenditures, though they decrease spending in both categories.

[28] A summary of the effect of the policy change predicted by the model without expenditures can be found in Epple et al. (2017), Table 3.

Interestingly, the lowest quality private school responds to the increase in federal aid in a fashion completely different from the other schools. While its spending was already shifted towards luxury expenditures more than any other school before the increase in federal aid, the increase in aid causes it to shift its spending towards luxury to an even greater extent. Moreover, it's average tuition price decreases more than 15% as it enrolls students with significantly lower incomes. Its quality drops below the quality at the state schools, but its per-student luxury expenditures are still higher than they are at the state schools. Since its tuition prices are also significantly lower than the other private schools, it remains an attractive option for students with high affinities for luxury.

The behavior of school 3 is especially troubling because of the large amount of federal aid used by its students. The bottom section of Table 4 shows that nearly 80% of the students at school 3 are receiving federal aid, and that more than 80% of those students are receiving the maximum amount. These students could receive a higher quality education for a significantly lower price by attending their state school. That being said, because these students choose to attend school 3, the school is able to offer large amounts of institutional aid to higher ability students who are too poor to afford college, even after the increase in the maximum aid amount.

While the behavior of the lowest quality private school is unexpected, the predictions of Proposition 3 are still upheld. The magnitude of the decrease in per-student luxury expenditures decreases as the school's price cap increases. Furthermore, the average price received at school 3 decreased, and its average student ability also decreased as it brought in new students with high effective marginal costs ($EMC(b)$), resulting in a significantly larger decrease in per-student luxury expenditures at school 3 than at the other schools.

In summary, with the exception of school 3, the model's predictions for the equilibrium effects of an increase in federal aid are consistent with the predictions made by the model in Epple et al. (2017). The top two private schools shift their spending away from luxury, increasing their educational expenditures and decreasing their luxury expenditures, as they rely less heavily on attracting high income students to subsidize the enrollment of low income students. Thus, they are able to increase their average student ability and increase their quality. The state schools see small decreases in their average student ability and quality, but these are considered to be acceptable given the increase in aggregate achievement resulting from the substantial increase in the number of students enrolled.

However, the behavior of the lowest quality private school sets the predictions of this model apart. While its behavior is unexpected, it is a result of the variation in students' affinities for luxury. Unable to effectively compete with the other private schools for the highest ability students, due to both its lower price cap and lower endowment, it lowers its prices and offers a more luxurious alternative to the state schools, for students with high affinities for luxury who either can't get into or can't afford the more luxurious private schools. By doing so, it is able to fund low income students who raise its average student ability.

The inclusion of expenditures allows for interesting features of product differentiation, and illuminates the sort of unintended consequences policies can have. The low quality private school receives substantially more federal aid than any of the other schools, but provides a lower quality education than the state schools, at a much higher price. From the perspective of trying to maximize achievement or welfare, this is clearly inefficient. Table 2 lists the total welfare and total achievement, for both the baseline equilibrium and after the aid limit is increased. Although total achievement increases after the increase in the aid limit, total achievement at school 3 actually decreases, and this is despite the increase in enrollment. Average achievement per student at school 3 drops by more than 7% after the increase in aid.

This is also true for total welfare. Total welfare at each school increases after the increase in aid, except for at school 3, where total welfare drops despite the increase in enrollment. In fact, total welfare for the entire market actually drops after the increase in aid, because the negative effect of the increased income taxes is not offset by the increase in utilities of students.

So we see that allowing schools to differentiate themselves through their expenditures generates an added dimension of competition that can result in inefficient outcomes. In the current analysis, increasing the the availability of federal aid increased the inefficient behavior.

Table 4: MaxAid=8 Values

j	k_j	θ_j	I_j	G_j	I_j/G_j	q_j	Ave.Tuit.	Avg.Inc.	Prop.Price Cap
A	0.1489	2.884	6.82	2.42	2.82	1.109	7.17	71.74	-
B	0.1489	2.884	6.82	2.42	2.82	1.109	7.17	71.74	-
3	0.0450	2.931	3.96	3.01	1.32	1.053	13.81	70.50	0.13%
4	0.0508	3.076	14.02	4.16	3.37	1.274	24.24	117.33	81.95%
5	0.0312	3.241	21.84	4.73	4.62	1.369	32.08	140.82	80.48%

j	Avg.Fed.Aid	% Rec.Aid	% Rec. $\overline{A}$	Avg.Aid Cond.	% Rec. $\overline{A}$ Cond.
A	2.82	43.4%	23.3%	6.50	53.7%
B	2.82	43.4%	23.3%	6.50	53.7%
3	5.92	79.4%	63.9%	7.45	80.5%
4	2.06	29.8%	18.8%	6.90	63.2%
5	1.55	22.8%	13.8%	6.82	60.6%

Table 5: Change (%) From Baseline to MaxAid=8

j	k_j	θ_j	I_j	G_j	I_j/G_j	q_j	Ave.Tuit.	Ave.Inc.	Prop.Price Cap
A	+6.3	-0.6	-0.7	-1.6	+1.1	-0.2	+1.1	-3.3	-
B	+6.3	-0.6	-0.7	-1.6	+1.1	-0.2	+1.1	-3.3	-
3	+5.1	-1.7	-32.5	-11.7	-23.3	-5.8	-15.4	-13.4	-1.3%
4	+9.0	+0.8	+5.5	-6.7	+13.1	+0.9	+0.5	-2.2	+6.1%
5	+2.3	+0.7	+1.7	-4.4	+6.5	+0.3	0.0	-2.7	-0.9%

j	Avg.Fed.Aid	% Rec.Aid	% Rec. $\overline{A}$	Avg.Aid Cond.	% Rec. $\overline{A}$ Cond.
A	+49.2	+4.7%	+0.9%	+32.9	-4.2%
B	+49.2	+4.7%	+0.9%	+32.9	-4.2%
3	+64.9	+14.8%	+11.3%	+34.2	-0.9%
4	+56.1	+4.5%	+1.4%	+31.9	-5.4%
5	+72.2	+5.3%	+2.2%	+31.9	-5.7%

6 Conclusions

This paper has presented a general equilibrium model of the market for higher education that includes competing state and private colleges with alternative objectives, students that differ by income, ability, affinity for luxury, and unobserved idiosyncratic preference for colleges, college qualities that depend on educational expenditures and student abilities, college luxury levels that depend on luxury expenditures, and federal aid modeled to approximate U.S. policy. The quantitative version of the model does a very good job of matching key characteristics of the U.S. market for higher education, including the distributions of students and financial aid across schools.

By implementing college expenditure decisions into the model, and allowing students to vary in their preference for luxuriousness, a new dimension of competition between schools is introduced, resulting in interesting and insightful model predictions. The results of the analysis suggest that when schools compete with each other using their expenditures, this can lead to inefficient outcomes. Furthermore, in certain conditions, increasing the availability of student aid may magnify the inefficiencies.

As discussed in the introduction, the relative success of a policy that increases federal student aid depends on the objective of the policy. For the given specification of the model, an increase in the maximum aid limit was demonstrated to have lowered total welfare in the market, resulting from the inefficient use of financial aid. Furthermore, while total achievement increased, it decreased at the lowest quality private school despite an increase in enrollment, demonstrating that the outcome was still inefficient.

Adding schools' expenditure choices to the model developed by Epple et al. (2017) has increased the model's ability to replicate empirical values. Furthermore, the extended model is able to simulate distributions of students across schools that do an excellent job of matching their empirically observed counterparts. The model presented in this paper provides a structure under which the equilibrium effects of financial aid policy can be more thoroughly analyzed, and it provides new insights into the way in which schools compete.

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Appendix A. Proofs of Propositions

Proof of Proposition 1:

Note that we have assumed that school quality $q_j(\cdot)$ is a twice differentiable, increasing, and strictly quasi-concave function of θ_j and I_j. We have also assumed that school j takes as given other schools' tuitions, qualities, and luxuries when solving its optimization problem. We now also assume that the problem is strictly quasi-concave, and we write out the Lagrangian:[29]

$$\begin{aligned} L = q + \lambda \left[\int\int\int \left(\sum_{s=1}^{S} \pi_s p_s r_s f_s \right) db\ dy\ dz + E - F - V - kI - kG \right] \\ + \eta \left[k\theta - \int\int\int \left(b \sum_{s=1}^{S} \pi_s r_s f_s \right) db\ dy\ dz \right] + \Omega \left[k - \int\int\int \left(\sum_{s=1}^{S} \pi_s r_s f_s \right) db\ dy\ dz \right] \end{aligned} \tag{33}$$

We now take the derivatives with respect to θ, I, G, and k, and find the first variation with respect to $p_s(b, y)$:

$$\begin{aligned} L_\theta = q_\theta + \lambda \int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial q} q_\theta f_s \right) db\ dy\ dz + \eta \left[k - \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_\theta f_s \right) db\ dy\ dz \right] \\ - \Omega \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_\theta f_s \right) db\ dy\ dz = 0. \end{aligned} \tag{34}$$

$$\begin{aligned} L_I = q_I + \lambda \left[\int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial q} q_I f_s \right) db\ dy\ dz - k \right] - \eta \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_I f_s \right) db\ dy\ dz \\ - \Omega \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_I f_s \right) db\ dy\ dz = 0. \end{aligned} \tag{35}$$

[29] I suppress all j subscripts and functional arguments.

$$L_G = \lambda \left[\int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial l} l_G f_s \right) db\ dy\ dz - k \right] - \eta \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} l_G f_s \right) db\ dy\ dz$$
$$-\Omega \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} l_G f_s \right) db\ dy\ dz = 0. \tag{36}$$

$$L_k = -\lambda \left[V' + I + G \right] + \eta\theta + \Omega = 0. \tag{37}$$

$$L_{p_s(b,y)} = \lambda \int \pi_s \left(r_s + p_s \frac{\partial r_s}{\partial p_s} \right) f_s dz - \eta \int \left(b\pi_s \frac{\partial r_s}{\partial p_s} f_s \right) dz - \Omega \int \left(\pi_s \frac{\partial r_s}{\partial p_s} f_s \right) dz = 0. \tag{38}$$[30]

Note that $q(\cdot)$ and $l(\cdot)$ are not functions of b, y, or z. Let:

$$A = \int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial q} f_s \right) db\ dy\ dz, \tag{39}$$

$$B = \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} f_s \right) db\ dy\ dz, \text{ and} \tag{40}$$

$$C = \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} f_s \right) db\ dy\ dz. \tag{41}$$

(42)

Then (34) and (35) can be rewritten as:

$$L_\theta = q_\theta + \lambda q_\theta A + \eta k - \eta q_\theta B - \Omega q_\theta C = 0, \text{ and} \tag{43}$$

$$L_I = q_I + \lambda q_I A - \lambda k - \eta q_I B - \Omega q_I C = 0. \tag{44}$$

Dividing (43) by q_θ and (44) by q_I, and setting them equal to each other, we get:

$$1 + \lambda A + \frac{\eta k}{q_\theta} - \eta B - \Omega C = 1 + \lambda A - \frac{\lambda k}{q_I} - \eta B - \Omega C \tag{45}$$

$$\frac{\eta k}{q_\theta} = -\frac{\lambda k}{q_I} \tag{46}$$

$$\frac{q_\theta}{q_I} = -\frac{\eta}{\lambda} \tag{47}$$

[30]Note that we now have $f(z|b, y)$.

Dividing (37) by λ, we get:

$$-\left[V' + I + G\right] + \left(\frac{\eta}{\lambda}\right)\theta + \frac{\Omega}{\lambda} = 0 \tag{48}$$

Substituting (47) into (48), we get:

$$-\left[V' + I + G\right] - \frac{q_\theta}{q_I}\theta = -\frac{\Omega}{\lambda} \tag{49}$$

Now divide (38) by $\left[\lambda \int \frac{\partial r_s}{\partial p_s} f_s dz\right]$:

$$\frac{\int r_s f_s dz}{\int \partial r_s / \partial p_s f_s dz} + p_s - \frac{\eta b}{\lambda} - \frac{\Omega}{\lambda} = 0 \tag{50}$$

Substitute (47) and (49) into (50) to get:

$$p_s + \frac{\int r_s f_s dz}{\int \partial r_s / \partial p_s f_s dz} + \frac{q_\theta}{q_I} b - \left[V' + I + G\right] - \frac{q_\theta}{q_I}\theta = 0 \tag{51}$$

$$p_s + \frac{\int r_s f_s dz}{\int \partial r_s / \partial p_s f_s dz} = V' + I + G + \frac{q_\theta}{q_I}(\theta - b) \tag{52}$$

Finally, adding notation back in and noting that this holds $\forall p_s(b, y)$, we get Proposition 1:

$$p_{sj}(b, y) + \frac{\int r_{sj}(b, y, z; \cdot) f_s(z|b, y) dz}{\int \left(\partial r_{sj}(b, y, z; \cdot) / \partial p_{sj}(b, y)\right) f_s(z|b, y) dz} = V'(k_j) + I_j + G_j + \frac{\partial q_j(\theta_j, I_j)/\partial \theta}{\partial q_j(\theta_j, I_j)/\partial I}(\theta_j - b),\ \forall p_s(b, y) \tag{53}$$

Proof of Proposition 2:

From the first-order conditions, one can write the first variation with respect to admission of in-state and out-of-state students as:

$$L_{\gamma_s} = \lambda \pi_s r_s f_s(b, y, z)[a(\cdot)/\lambda + T_s + w - EMC_s(b)] \tag{54}$$

$$L_{\gamma_{so}} = \lambda \left(\sum_{t \neq s} \pi_t r_{ts} f_t(b, y, z) \right) [T_{so} + w - EMC_s(b)] \tag{55}$$

where $\lambda > 0$ is the Lagrange multiplier associated with the budget constraint in Eq.(14). Now, using the feasibility constraints

$$\gamma_s(b, y), \gamma_{so}(b, y) \in [0, 1] \text{ for all students } (t, b, y), \tag{56}$$

we obtain the results.

Proof of Proposition 3:

I will prove the private school's problem first. Begin with the first order condition for G:

$$\begin{aligned} L_G = \lambda \left[\int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial l} l_G f_s \right) db\ dy\ dz - k \right] - \eta \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} l_G f_s \right) db\ dy\ dz \\ - \Omega \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} l_G f_s \right) db\ dy\ dz = 0. \end{aligned} \tag{57}$$

$$\begin{aligned} \implies l_G \Bigg\{ \lambda \left[\int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz - \frac{k}{l_G} \right] - \eta \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz \\ - \Omega \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz \Bigg\} = 0. \end{aligned} \tag{58}$$

$$\implies \lambda \left[\int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz - \frac{k}{l_G} \right] - \eta \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz$$
$$- \Omega \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz = 0. \tag{59}$$

$$\implies \int\int\int \left(\sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz - \frac{k}{l_G} - \frac{\eta}{\lambda} \int\int\int \left(b \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz$$
$$- \frac{\Omega}{\lambda} \int\int\int \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz = 0. \tag{60}$$

$$\implies \int\int\int p \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz - \frac{k}{l_G} - \int\int\int \left(\frac{\eta}{\lambda} b + \frac{\Omega}{\lambda} \right) \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz = 0. \tag{61}$$

$$\implies \int\int\int [p - EMC(b)] \left(\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial l} f_s \right) db\ dy\ dz = \frac{k}{l_G}. \tag{62}$$

For the parameterization given in (24)-(30), we have that $l(G) = G^\delta$, so $l_G = \delta G^{\delta-1}$. And we also have that

$$\frac{\partial r_j}{\partial l_j} = \frac{\alpha \left[(y - p_j - S + A_j(y))\, q_j l_j \right]^{\alpha-1} (y - p_j - S + A_j(y))\, q_j}{\sum_{k \in J} \left[(y - p_k - S + A_k(y))\, q_k l_k \right]^{\alpha}}$$
$$+ \frac{\left[(y - p_j - S + A_j(y))\, q_j l_j \right]^{\alpha} (-1)\alpha \left[(y - p_j - S + A_j(y))\, q_j l_j \right]^{\alpha-1} (y - p_j - S + A_j(y))\, q_j}{\left[\sum_{k \in J} \left[(y - p_k - S + A_k(y))\, q_k l_k \right]^{\alpha} \right]^2}$$
$$= \frac{\alpha r_j}{l_j} - \frac{\alpha r_j r_j}{l_j}$$
$$= \frac{\alpha r_j (1 - r_j)}{l_j}, \tag{63}$$

So we can rewrite (62) as

$$\implies \frac{\alpha}{G^{\delta}} \int\int\int [p - EMC(b)] \left(\sum_{s=1}^{S} \pi_s r_s (1 - r_s) f_s \right) db\ dy\ dz = \frac{k}{\delta G^{\delta - 1}}. \tag{64}$$

$$\implies G = \frac{\alpha\delta}{k} \int\int\int [p - EMC(b)] \left(\sum_{s=1}^{S} \pi_s r_s (1 - r_s) f_s \right) db\ dy\ dz \tag{65}$$

$$\implies G_j = \frac{\alpha\delta}{k_j} \int\int\int \left(\sum_{s=1}^{S} \pi_s \left[p_{sj} - EMC_j(b) \right] r_{sj} (1 - r_{sj}) f_s(b, y, z) \right) db\ dy\ dz. \tag{66}$$

Now I will prove the public school expression. Start with the first order condition for G:

$$\begin{aligned} L_G = & \int\int\int \left(a\pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} l_G f_s \right) db\ dy\ dz \\ & + \lambda \left[\int\int\int \pi_s p_{ss} \gamma_s \frac{\partial r_{ss}}{\partial l} l_G f_s db\ dy\ dz + \int\int\int \gamma_{so} \left(\sum_{t \neq s} \pi_t p_{ts} \frac{\partial r_{ts}}{\partial l} l_G f_t \right) db\ dy\ dz - k_s \right] \\ & - \eta \left[\int\int\int b\pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} l_G f_s db\ dy\ dz + \int\int\int b\gamma_{so} \left(\sum_{t \neq s} \pi_t \frac{\partial r_{ts}}{\partial l} l_G f_t \right) db\ dy\ dz \right] \\ & - \Omega \left[\int\int\int \pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} l_G f_s db\ dy\ dz + \int\int\int \gamma_{so} \left(\sum_{t \neq s} \pi_t \frac{\partial r_{ts}}{\partial l} l_G f_t \right) db\ dy\ dz \right] = 0. \end{aligned} \tag{67}$$

$$
\begin{aligned}
\Longrightarrow l_G \Bigg\{ & \int\int\int \Big(a\pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} f_s \Big) db\ dy\ dz \\
& + \lambda \Bigg[\int\int\int \pi_s p_{ss} \gamma_s \frac{\partial r_{ss}}{\partial l} f_s db\ dy\ dz + \int\int\int \gamma_{so} \Bigg(\sum_{t \neq s} \pi_t p_{ts} \frac{\partial r_{ts}}{\partial l} f_t \Bigg) db\ dy\ dz - \frac{k_s}{l_G} \Bigg] \\
& - \eta \Bigg[\int\int\int b\pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} f_s db\ dy\ dz + \int\int\int b\gamma_{so} \Bigg(\sum_{t \neq s} \pi_t \frac{\partial r_{ts}}{\partial l} f_t \Bigg) db\ dy\ dz \Bigg] \\
& - \Omega \Bigg[\int\int\int \pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} f_s db\ dy\ dz + \int\int\int \gamma_{so} \Bigg(\sum_{t \neq s} \pi_t \frac{\partial r_{ts}}{\partial l} f_t \Bigg) db\ dy\ dz \Bigg] \Bigg\} = 0.
\end{aligned} \tag{68}
$$

$$
\begin{aligned}
\Longrightarrow \frac{1}{\lambda} & \int\int\int \Big(a\pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} f_s \Big) db\ dy\ dz \\
& + \Bigg[\int\int\int \pi_s p_{ss} \gamma_s \frac{\partial r_{ss}}{\partial l} f_s db\ dy\ dz + \int\int\int \gamma_{so} \Bigg(\sum_{t \neq s} \pi_t p_{ts} \frac{\partial r_{ts}}{\partial l} f_t \Bigg) db\ dy\ dz - \frac{k_s}{l_G} \Bigg] \\
& - \frac{\eta}{\lambda} \Bigg[\int\int\int b\pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} f_s db\ dy\ dz + \int\int\int b\gamma_{so} \Bigg(\sum_{t \neq s} \pi_t \frac{\partial r_{ts}}{\partial l} f_t \Bigg) db\ dy\ dz \Bigg] \\
& - \frac{\Omega}{\lambda} \Bigg[\int\int\int \pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} f_s db\ dy\ dz + \int\int\int \gamma_{so} \Bigg(\sum_{t \neq s} \pi_t \frac{\partial r_{ts}}{\partial l} f_t \Bigg) db\ dy\ dz \Bigg] = 0.
\end{aligned} \tag{69}
$$

$$
\begin{aligned}
\Longrightarrow \ \frac{1}{\lambda} & \int\int\int \Big(a\pi_s \gamma_s \frac{\partial r_{ss}}{\partial l} f_s \Big) db\ dy\ dz \\
& + \int\int\int \pi_s \left[p_{ss} - \left(\frac{\eta}{\lambda} b + \frac{\Omega}{\lambda} \right) \right] \gamma_s \frac{\partial r_{ss}}{\partial l} f_s db\ dy\ dz \\
& + \int\int\int \left[p_{ts} - \left(\frac{\eta}{\lambda} b + \frac{\Omega}{\lambda} \right) \right] \gamma_{so} \Bigg(\sum_{t \neq s} \pi_t \frac{\partial r_{ts}}{\partial l} f_t \Bigg) db\ dy\ dz - \frac{k_s}{l_G} = 0.
\end{aligned} \tag{70}
$$

$$\implies \frac{1}{\lambda}\int\int\int\Big(a\pi_s\gamma_s\frac{\partial r_{ss}}{\partial l}f_s\Big)db\ dy\ dz$$
$$+\int\int\int \pi_s\left[p_{ss}+w_s-EMC_s(b)\right]\gamma_s\frac{\partial r_{ss}}{\partial l}f_s db\ dy\ dz \tag{71}$$
$$+\int\int\int \left[p_{ts}+w_s-EMC_s(b)\right]\gamma_{so}\left(\sum_{t\neq s}\pi_t\frac{\partial r_{ts}}{\partial l}f_t\right)db\ dy\ dz=\frac{k_s}{l_G}.$$

$$\implies \frac{\alpha}{G\lambda}\int\int\int\Big(a\pi_s\gamma_s r_{ss}(1-r_{ss})f_s\Big)db\ dy\ dz$$
$$+\frac{\alpha}{G}\int\int\int \pi_s\left[p_{ss}+w_s-EMC_s(b)\right]\gamma_s r_{ss}(1-r_{ss})f_s db\ dy\ dz$$
$$+\frac{\alpha}{G}\int\int\int \left[p_{ts}+w_s-EMC_s(b)\right]\gamma_{so}\left(\sum_{t\neq s}\pi_t r_{ts}(1-r_{ts})f_t\right)db\ dy\ dz=\frac{k_s}{\delta G^{\delta-1}}. \tag{72}$$

$$\implies G=\frac{\delta\alpha}{k_s\lambda}\int\int\int\Big(a\pi_s\gamma_s r_{ss}(1-r_{ss})f_s\Big)db\ dy\ dz$$
$$+\frac{\delta\alpha}{k_s}\int\int\int \pi_s\left[p_{ss}+w_s-EMC_s(b)\right]\gamma_s r_{ss}(1-r_{ss})f_s db\ dy\ dz$$
$$+\frac{\delta\alpha}{k_s}\int\int\int \left[p_{ts}+w_s-EMC_s(b)\right]\gamma_{so}\left(\sum_{t\neq s}\pi_t r_{ts}(1-r_{ts})f_t\right)db\ dy\ dz. \tag{73}$$

$$\implies G=\frac{\delta\alpha}{k_s}\Bigg[\frac{1}{\lambda}\int\int\int\Big(a\pi_s\gamma_s r_{ss}(1-r_{ss})f_s\Big)db\ dy\ dz$$
$$+\int\int\int \pi_s\left[p_{ss}+w_s-EMC_s(b)\right]\gamma_s r_{ss}(1-r_{ss})f_s db\ dy\ dz$$
$$+\int\int\int \left[p_{ts}+w_s-EMC_s(b)\right]\gamma_{so}\left(\sum_{t\neq s}\pi_t r_{ts}(1-r_{ts})f_t\right)db\ dy\ dz\Bigg]\blacksquare \tag{74}$$

Given the specification in (12)-(18), we have choice probabilities of the form:

$$r_j\left(b, y, z; P(b, y), Q, L\right) = \frac{\left[\left(y - p_j - S + A_j(y)\right) q_j l_j\right]^{\alpha}}{\sum\limits_{k \in J} \left[\left(y - p_k - S + A_k(y)\right) q_k l_k\right]^{\alpha}}. \tag{75}$$

Taking the derivative with respect to q_j, we get:

$$\begin{aligned}
\frac{\partial r_j}{\partial q_j} &= \frac{\alpha\left[\left(y - p_j - S + A_j(y)\right) q_j l_j\right]^{\alpha-1}\left(y - p_j - S + A_j(y)\right) l_j}{\sum_{k \in J}\left[\left(y - p_k - S + A_k(y)\right) q_k l_k\right]^{\alpha}} \\
&+ \frac{\left[\left(y - p_j - S + A_j(y)\right) q_j l_j\right]^{\alpha}(-1)\alpha\left[\left(y - p_j - S + A_j(y)\right) q_j l_j\right]^{\alpha-1}\left(y - p_j - S + A_j(y)\right) l_j}{\left[\sum_{k \in J}\left[\left(y - p_k - S + A_k(y)\right) q_k l_k\right]^{\alpha}\right]^2} \\
&= \frac{\alpha r_j}{q_j} - \frac{\alpha r_j r_j}{q_j} \\
&= \frac{\alpha r_j (1 - r_j)}{q_j}
\end{aligned} \tag{76}$$

Taking the derivative with respect to l_j, we get:

$$\begin{aligned}
\frac{\partial r_j}{\partial l_j} &= \frac{\alpha\left[\left(y - p_j - S + A_j(y)\right) q_j l_j\right]^{\alpha-1}\left(y - p_j - S + A_j(y)\right) q_j}{\sum_{k \in J}\left[\left(y - p_k - S + A_k(y)\right) q_k l_k\right]^{\alpha}} \\
&+ \frac{\left[\left(y - p_j - S + A_j(y)\right) q_j l_j\right]^{\alpha}(-1)\alpha\left[\left(y - p_j - S + A_j(y)\right) q_j l_j\right]^{\alpha-1}\left(y - p_j - S + A_j(y)\right) q_j}{\left[\sum_{k \in J}\left[\left(y - p_k - S + A_k(y)\right) q_k l_k\right]^{\alpha}\right]^2} \\
&= \frac{\alpha r_j}{l_j} - \frac{\alpha r_j r_j}{l_j} \\
&= \frac{\alpha r_j (1 - r_j)}{l_j}
\end{aligned} \tag{77}$$

Appendix B. Data

This section provides information about the data used to analyze empirical trends in the market for higher education. The data used is taken from the Delta Cost Project database. The National Center for Education Statistics (NCES) maintains the Delta Cost Projects database as part of its Integrated Postsecondary Education Data System (IPEDS). IPEDS conducts annual surveys gathering information from every college, university, and technical and vocational institution that participates in the federal student financial aid programs.

The Delta Cost Project reports the expenditures and revenues of colleges, broken down into several different categories. While there are no expenditure categories that perfectly capture the educational and luxury expenditures of schools, there are categories that capture the essence of what I am looking for, and these categories serve as proxies in the analysis.

As a proxy for educational expenditures, I use each school's reported expenditures on instruction. The following is the definition for instructional expenditures given in the Data Dictionary for the Delta Cost Project database:

> *Instruction - A functional expense category that includes expenses of the colleges, schools, departments, and other instructional divisions of the institution and expenses for departmental research and public service that are not separately budgeted. Includes general academic instruction, occupational and vocational instruction, community education, preparatory and adult basic education, and regular, special, and extension sessions. Also includes expenses for both credit and non-credit activities. Excludes expenses for academic administration where the primary function is administration (e.g., academic deans). Information technology expenses related to instructional activities are included if the institution separately budgets and expenses information technology resources (otherwise these expenses are included in academic support). Operations and maintenance and interest amounts attributed to the instruction function have been subtracted from the total instructional expenditure amount at FASB reporting institutions. Operations and maintenance amounts (and interest in the aligned form beginning in 2009) attributed to the instruction function have been subtracted from the total amount at public Aligned form reporting institutions.*

As a proxy for luxury expenditures, I use each school's reported auxiliary expenditures, which covers expenditures for items such as student dormitories, recreational centers, and student union buildings. The following is the definition given for auxiliary expenditures in the Data Dictionary:

> *Auxiliary enterprises - total expenses is the sum of all operating expenses associated with essentially self-supporting operations of the institution that exist to furnish a service to students, faculty, or staff, and that charge a fee that is*

directly related to, although not necessarily equal to, the cost of the service. Examples are residence halls, food services, student health services, intercollegiate athletics (only if essentially self-supporting), college unions, college stores, faculty and staff parking, and faculty housing. The amount of interest attributed to the auxiliary enterprise function has been subtracted from the total auxiliary enterprise expenditure amount at institutions reporting on the FASB or aligned form (beginning in 2009).

In order to estimate per-student expenditures, I divide total expenditures for each school by the reported Full Time Equivalent (FTE) student count. FTE is estimated using weights for part-time students. The following is the definition for FTE student count given in the Data Dictionary:

Full-time equivalent enrollments are derived from the enrollment by race/ethnicity section of the fall enrollment survey. The full-time equivalent of an institution's part-time enrollment is estimated by multiplying part-time enrollment by factors that vary by control and level of institution and level of student; the estimated full-time equivalent of part-time enrollment is then added to the full-time enrollment of the institution. This formula is used by the U.S. Department of Education to produce the full-time equivalent enrollment data published annually in the Digest of Education Statistics.

For most of the cross-sectional analysis, observations are limited to academic year 2007, since that is the year for which the model is calibrated. In those cases, dollar values are left in their original form and are not adjusted for inflation. For instances where trends over time are analyzed, all dollar values are normalized to 2013 dollars using the CPI-U scalar provided in the Delta Cost Project Database.

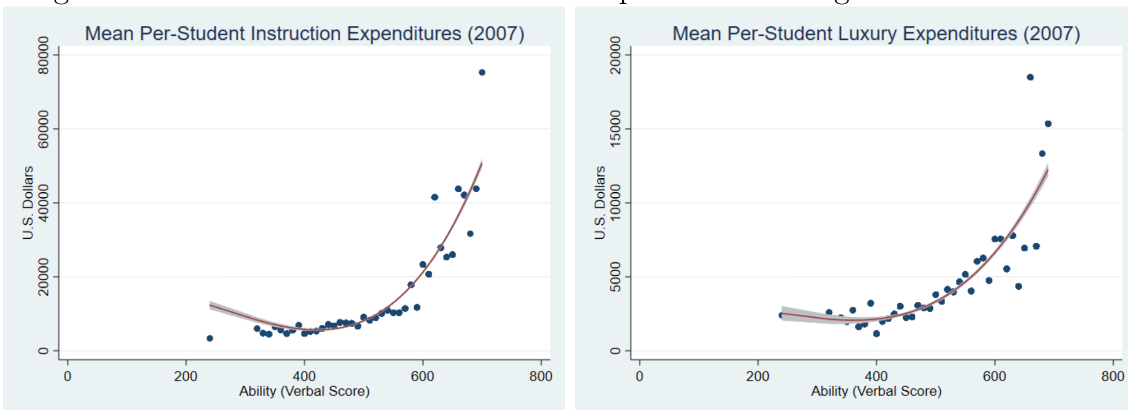
Whenever averages are calculated (e.g. average per-student expenditures), they are calculated using a weighted average of all schools, weighted by the reported FTE student count.

Figures 4 and 5

Figure 4 Correlation = 0.6588; Figure 5 Correlation = 0.5785

As a proxy for average student ability at schools, I use each school's reported 25th percentile score on the quantitative section of the SAT. While both the 25th and 75th percentile scores are reported, I use the 25th percentile scores because they have greater variation across schools, since the 75th percentile scores bunch at 800 for many of the top schools. Scores are also reported for the verbal section of the SAT. Using verbal scores in place of quantitative scores has little effect on the observed patterns. Figures B.1 and B.2 below recreate Figures 4 and 5, respectively, using the 25th percentile scores for the veral section of the SAT.

Figures B.1 and B.2: Mean Per-Student Expenditures Using SAT Verbal Scores



As can be seen, the exponential nature of per-student expenditures is still prevalent. However, the distributions are more compact.

To plot the relationship between average ability and expenditures, I round each school's average SAT score to the nearest 10 point value. Then, for each score, the average per-student expenditures are calculated, weighting each school by its FTE student count as detailed above.

The distribution of points is then fit with a quadratic curve, with the 95 percent CI shown in grey.

Figures 6 and 7

Figure 6 Correlation = 0.5967;   Figure 7 Correlation = 0.5701

To plot the relationships between tuition prices and expenditures, I round the tuition prices of each school to the nearest $1000. Then, taking all schools with the same rounded tuition price, I calculate the average per-student expenditures. As before, the average is weighted by the number of FTE students at each school.

Figures 8 and 9

Figure 8 Correlation = 0.3969;   Figure 9 Correlation = 0.1455

Figures 8 and 9 are created using the same strategy, but for school size instead of price. The reported FTE student count for each school is rounded to the nearest 1000 students, and schools are grouped by their rounded student counts. Taking all schools with the same rounded student count, weighted averages are calculated for per-student expenditures, weighting by the true (unrounded) FTE student count at each school.

Figures 12 and 13

To create Figure 12, each school's 25th percentile score on the quantitative section of the SAT is rounded to the nearest 10 point value. Then, for each 10 point value, the total number of FTE students attending schools with that score is summed.

Similarly for Figure 13, each school's tuition price is rounded to the nearest $1000. Then, for each $1000 value, the total number of FTE students attending schools with that tuition price is summed. For some reason there is a spike in the number of students attending schools with a tuition price that rounds to $33,000. It is unclear why tuition prices appear to collect at this value.

Calibrating Average Expenditures

I need to calibrate the average per-student instructional expenditures at public schools, the average per-student auxiliary expenditures at public schools, the average per-student instructional expenditures at private schools, and the average per-student auxiliary expenditures at private schools.

Instructional expenditures are calculated using the variable instruction01_fte. Auxiliary expenditures are calculated using auxiliary01_fte. In all cases, only observations for academicyear 2007 are considered. Public and private schools are considered individually by using the variable sector_revised.

When calculating the average per-student expenditures, the weighted average is taken, weighting by the total number of Full Time Equivalent (FTE) students, which is estimated using the variable fte_count.